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* In Kazan University the Electron Paramagnetic Resonance (EPR) was discovered by Zavoisky E.K. in 1944.

Impurity spin in normal stochastic field: basic model of magnetic resonance†

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Famous Anderson-Weiss-Kubo model of magnetic resonance is reconsidered in order to bridge existing gaps in its applications for solutions of fundamental problems of spin dynamics and theory of master equations. The model considers the local field fluctuations as one-dimensional normal random process. We refined the conditions of applicability of perturbation theory to calculate the spin depolarization. It is shown that for very slow fluctuations the behavior of the longitudinal magnetization is simply related to the correlation function of the local field. The effect could be checked by the experimental studies of magnetic resonance in quasi-Ising paramagnets.

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Spin system with the Hamiltonian (written in rotating frame)

$$H = H_0(t) + H_1, \quad H_1 = \omega_1 I_x, \qquad H_0(t) = \left(\Delta + \omega_l(t)\right) I_z = H_I + H_l(t) \tag{1}$$

is one of most important basic models for studies in spin dynamics. Here I_α is spin operator, Δ is the detuning from the resonance, ω_1 represents magnitude of the rotating field, and $\omega_l(t)$ corresponds to time dependent local field, produced by surrounding substance. Famous Anderson-Weiss-Kubo (AWK) model [1] considers $\omega_l(t)$ as a normal stationary stochastic process with the correlation function $\left\langle \omega_l(t)\omega_l(t_1)\right\rangle_n = M_2 \kappa\left(\left|t-t_1\right|\right), \quad M_2 = \left\langle \omega_l^2\right\rangle_n$, it means, that for any reasonable function $\alpha(t)$ the moment-generating functional is of the form

$$\Phi(t,t_1,[\alpha]) = \left\langle \exp\left(i\int_{t_1}^{t} d\tau \alpha(\tau)\omega_l(\tau)\right)\right\rangle_n = \exp\left(-\frac{M_2}{2}\int_{t_1}^{t} d\tau_1 d\tau_2 \alpha(\tau_1)\alpha(\tau_2)\kappa(\tau_1-\tau_2)\right). \tag{2}$$

Here averaging is fulfilled on distribution of random trajectories $\omega_l(t)$.

The model was created to explain the “narrowing of the resonance line by motion” using Gaussian or simple exponential $\kappa\left(\left|t\right|\right)$, but it was successful for explaining of the line shape for impurity beta-active nuclei [2, 3] without observable differences between precise experimental data and theoretical predictions for more realistic $\kappa\left(\left|t\right|\right)$. It was adopted to describe two- and multi-spin transitions [3-5] with successful incorporation both static and dynamic correlations of local fields on impurity spins [5, 6]. The model found important application in description of the electron spin echo [7] and it produces a kernel for modern theory of spin dynamics in magnetically diluted systems [8, 9].

An important application of the AWK model consists in derivation of the applicability conditions for perturbation theory for small ω_1 in calculation of longitudinal correlation function [5, 10, 11]

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$$F(t) = \left\langle I_z I_z(t) \right\rangle / \left\langle I_z^2 \right\rangle = \left\langle \left\langle I_z I_z(t) \right\rangle_0 \right\rangle_n / \left\langle I_z^2 \right\rangle_0 . \tag{3}$$

Here $\left\langle A \right\rangle_0 = Tr\left(A\right)/Tr\left(1\right)$ for any relevant operator A, that corresponds to averaging with infinite spin temperature. The function $F(t)$ is proportional to observable value of the operator $I_z(t)$, if the initial state of the density matrix $\rho(t)$ is of standard form

$$\rho_0 = \rho(t=0) = \frac{1}{Tr(1)}\left(1 + \frac{3p_0}{I(I+1)} I_z\right), \qquad p_0 = Tr\left(I_z \rho(t=0)\right). \tag{4}$$

Indeed, p_0 doesn't depend on ω_l, therefore

$$J_z(t) = \left\langle Tr\left(I_z \rho(t)\right)\right\rangle_n = \left\langle Tr\left(I_z(t) \rho(t=0)\right)\right\rangle_n = F(t) p_0 . \tag{5}$$

The advantage of the model is the realistic smooth time dependence of local field contrary to known exactly solvable models with hopping evolution of $\omega_l(t)$. It is usually expected [1, 10, 11], that if $\Delta = 0$, then the simplest conditions $\varepsilon_1 = R_0 T_2 \ll 1$, and $\varepsilon_2 = R_0 \tau_c \ll 1$ produce $F(t) = \exp(-R_0 t)$. Here

$$R_0 = \omega_1^2 T_2, \quad T_2 = \int_0^\infty dt \exp\left(-M_2 \int_0^t d\tau (t-\tau)\kappa(\tau)\right), \quad \tau_c = \int_0^\infty dt \kappa(t), \tag{6}$$

Nothing is known for slow smooth motion, when $R_0 \tau_c \gg 1$.

We will indicate below, that for smooth $\omega_l(t)$ there exist logarithmical correction to the condition with ε_2, similar to the correction, indicated previously for two-spin flip-flop transitions [5]. Further we will construct the solution for very slow smooth motions, which is valid in the main order in $\varepsilon_1 = \omega_1^2 T_2^2 \ll 1$. It has the form

$$F(t) = \frac{2}{\pi} \arcsin \kappa(t).$$

1. Very fast fluctuations – δ-correlated local fields

For the sake of brevity below $t > 0$. If $\tau_c = 0$ then correlation of local field can be written as

$$\left\langle \omega_l(t) \omega_l(t_1) \right\rangle_n = \frac{2}{T_2} \delta\left(t - t_1\right), \tag{7}$$

and the phase evolutions at different times are independent. It means that for $t \le t_1 \le t'$ we have

$$\Phi(t,t',[\alpha]) = \Phi(t,t_1,[\alpha]) \Phi(t_1,t',[\alpha]). \tag{8}$$

The quantum Liouville equation for density matrix $\rho(t)$

$$\frac{\partial}{\partial t}\rho = -i\left[H,\rho\right] = -iL\rho = -i\left(L_0 + L_1\right)\rho, \tag{9}$$

$$L_0 \rho = \left[H_0, \rho\right], \quad L_1 \rho = \left[H_1, \rho\right],$$

can be rewritten in the integral form

$$\rho(t) = e^{-i\int_0^t d\tau L_0(\tau)} \rho_0 - i\int_0^t d\tau e^{-i\int_\tau^t d\tau_1 L_0(\tau_1)} L_1 \rho(\tau). \tag{10}$$

Here and below we will use superoperator formalism, basic information about which can be found, for example, in the textbook [12]. The superoperator $L_0(t)$ commutes with itself for different times

$L_0(t_1)L_0(t_2) = L_0(t_2)L_0(t_1)$, that admits to use simple exponential in (10) instead of chronological ordering, required in absence of the commutativity, and, as a consequence, the multipliers with phase evolution in (10) can be calculated exactly:

$$U(t-t_1) = \left\langle \exp\left(-i\int_{t_1}^{t} d\tau L_0(\tau)\right)\right\rangle_n = \exp\left(-\left(i\Delta I_z^{\times} + \frac{1}{T_2}\left(I_z^{\times}\right)^2\right)\left|t-t_1\right|\right), \tag{11}$$

New superoperator $I_z^{\times}$ here is produced from usual spin operator I_z according to the standard rule $I_z^{\times} f = \left[I_z, f\right]$, where f is arbitrary operator.

Averaging the Eq. (10) we have, as in Ref. [13],

$$\left\langle \rho(t)\right\rangle_n = \left\langle \exp\left(-i\int_0^t d\tau L_0(\tau)\right)\right\rangle_n \rho_0 - i\int_0^t dt_1 \left\langle \exp\left(-i\int_{t_1}^t d\tau L_0(\tau)\right)\right\rangle_n L_1 \left\langle \rho(t_1)\right\rangle. \tag{12}$$

The relation (8) together with $\tau > t_1$ was applied here.

As a result of Eqs. (11) and (12) we have for $t > 0$

$$\frac{\partial}{\partial t}\left\langle \rho(t)\right\rangle_n = -\left(i\Delta I_z^{\times} + R\left(I_z^{\times}\right)^2 + iL_1\right)\left\langle \rho(t)\right\rangle_n. \tag{13}$$

Last equation is equivalent to Bloch's equations in absence of longitudinal spin-lattice relaxation

$$\frac{\partial}{\partial t}J_z = \left[\mathbf{\Omega}\times\mathbf{J}\right]_z, \quad \frac{\partial}{\partial t}J_{x,y} = \left[\mathbf{\Omega}\times\mathbf{J}\right]_{x,y} - \frac{1}{T_2}J_{x,y}, \quad \mathbf{\Omega} = \left(\omega_1, 0, \Delta\right), \tag{14}$$

where $J_\alpha(t) = Tr\left(I_\alpha \left\langle \rho(t)\right\rangle_n\right)$ are average values of spin operators. Excluding orthogonal components J_x and J_y with initial condition $J_{x,y}(t=0)$ we arrive to the equation for polarization along *z*-axis

$$\frac{\partial}{\partial t}J_z = -\omega_1^2 \,\mathrm{Re}\int_0^t d\tau e^{\left(i\Delta - \frac{1}{T_2}\right)\tau} J_z(t-\tau), \tag{15}$$

which is exact for δ-correlated process (7). We see that for small ω_1 the derivative $\partial J_z/\partial t \sim \omega_1^2$ is small. Therefore the variation of $J_z(t-\tau)$ during the times τ, important for the factor $\exp\left(\left(i\Delta - 1/T_2\right)\tau\right)$ in the integrand of Eq. (15), is negligible, and for $t > T_2$ we can replace the Eq. (15) by

$$\frac{\partial}{\partial t}J_z = -\omega_1^2 \,\mathrm{Re}\int_0^\infty d\tau e^{\left(i\Delta - \frac{1}{T_2}\right)\tau} J_z(t) = -R(\Delta)J_z(t). \qquad R(\Delta) = \pi\omega_1^2 g(\Delta), \tag{16}$$

$$g(\Delta) = \mathrm{Re}\int_{-\infty}^{\infty}\frac{dt}{2\pi}e^{i\Delta t - |t|/\mathrm{T}_2} = \frac{T_2}{\pi\left(1+\Delta^2 T_2^2\right)}.$$

Here normalized resonance line shape $g(\Delta)$ is introduced; it is a Fourier transform of free induction decay, which has simple exponential form $F_0(t) = \exp\left(-|t|/T\right)$ for δ-correlated local field.

Comparing Eqs. (15) and (16) we see, that ω_1 is small, if, at least, $R \ll 1/T_2$. To refine the condition we can retain in (16) next, linear in τ term of $J_z(t-\tau)$ expansion

$$\frac{\partial}{\partial t}J_z = -\omega_1^2 \,\mathrm{Re}\int_0^\infty d\tau e^{\left(i\Delta - \frac{1}{T_2}\right)\tau}\left(J_z(t) - \tau\frac{\partial}{\partial t}J_z(t)\right) = -Rt + \vartheta\frac{\partial}{\partial t}J_z, \tag{17}$$

$$\vartheta = \mathrm{Re}\,\omega_1^2\int_0^\infty d\tau e^{\left(i\Delta - \frac{1}{T_2}\right)\tau}\tau = \omega_1^2\frac{T_2^2\left(1-\left(\Delta T_2\right)^2\right)}{\left(1+\left(\Delta T_2\right)^2\right)^2}, \quad |\vartheta| \le R(\Delta)T_2 \le R_0 T_2 = \varepsilon_1, \quad R_0 = R\left(\Delta = 0\right).$$

It is evident, that condition of smallness of ω_1 received the form $|\theta| \ll 1$. For $\Delta = 0$ new condition coincides with previous $\varepsilon_1 = R_0 T_2 = \omega_1^2 T_2^2 \ll 1$, but with increasing of Δ it can be less restrictive.

2. Fast fluctuation of local fields and the perturbation theory

Modern perturbation theory consists of two different, but connected parts: obtaining the effective Hamiltonian and derivation of master equation, see [14] for example. Master equation is an equation for important part of the density matrix $\rho(t)$, which is sufficient for calculation of necessary observables. To derive it according to projection technique of Nakajima-Zwanzig we can introduce the projection superoperator P, which separates the important part:

$$P\rho = \langle \rho_D \rangle_n = \left(\langle \rho \rangle_n \right)_D . \tag{18}$$

Here index D separates the part, diagonal in representation of eigenstates of I_z, i.e., if $I_z |m\rangle = m|m\rangle$, then $\langle n|\rho_D|m\rangle = \delta_{mn} \langle m|\rho|m\rangle$.

Multiplication of Liouville Eq.(9) on P and $\overline{P} = 1 - P$ produces

$$\frac{\partial}{\partial t} P\rho = -iPL\left(P + \overline{P}\right)\rho, \quad \frac{\partial}{\partial t} \overline{P}\rho = -i\overline{P}L\left(P + \overline{P}\right)\rho.$$

Solving second equation with initial condition $\overline{P}\rho_0 = 0$ and substituting the solution into the first equation, we receive a master equation

$$\frac{\partial}{\partial t} P\rho = -\int_0^t d\tau M(\tau) P\rho(t-\tau), \tag{19}$$

$$M(\tau) = PL_1 T \exp\left(-i\int_0^\tau ds \overline{P} L(s) \overline{P}\right) L_1 P.$$

Here $T\exp(\cdots)$ is the standard chronological exponential. It is taken into account here, that the projectors (18) obey the relations $PLP = 0$, $PL\overline{P} = PL_1\overline{P} = PL_1$, $\overline{P}LP = \overline{P}L_1 P = L_1 P$. The memory kernel $M(t)$ has second order in $L_1 \sim \omega_1$. As a consequence the main order master equation is of the form

$$\frac{\partial}{\partial t} P\rho = -\int_0^t d\tau \left(\left\langle L_1 \exp\left(-i\int_\tau^t ds L_0(s)\right) L_1 \right\rangle_n P\rho(\tau) \right)_D = -\int_0^t d\tau M_0(t-\tau) P\rho(\tau). \tag{20}$$

It is taken into account here, that $L_0(t_1)L_0(t_2) = L_0(t_2)L_0(t_1)$ (therefore Texp is not necessary), and action of L_0 on nondiagonal operators produces nondiagonal operator as well (therefore $\overline{P}$ is not necessary).

Substituting here $L_0(t) = \omega_l(t) I_z^\times$ and $L_1 = \omega_1 I_x^\times$, after straightforward transformations we obtain that the polarization satisfies the equation

$$\frac{\partial J_z(t)}{\partial t} = -\frac{\omega_1^2}{4}\int_0^t d\tau Tr\left\{ \left(\left[[I_z, I_+], I_- \right] \left\langle e^{i\Delta(t-\tau)+i\varphi(t,\tau)} \right\rangle_n + \left[[I_z, I_-], I_+ \right] \left\langle e^{-i\Delta(t-\tau)-i\varphi(t,\tau)} \right\rangle_n \right) P\rho(\tau) \right\},$$

where $\varphi(t,\tau) = \int_\tau^t ds\omega_l(s)$. The function $F_0(t-\tau) = \left\langle \exp\left(i\varphi(t,\tau)\right) \right\rangle_n = \left\langle \exp\left(-i\varphi(t,\tau)\right) \right\rangle_n$ represents, evidently, free induction decay.

Now, after calculation of the commutators, we have

$$\frac{\partial J_z(t)}{\partial t} = -\omega_1^2 \int_0^t d\tau \cos\left(\Delta(t-\tau)\right) F_0(t-\tau) J_z(\tau) = -\int_0^t d\tau\, W_0(\tau) J_z(t-\tau), \tag{21}$$

$$W_0(\tau)=\omega_1^2\cos(\Delta\tau)F_0(\tau)=Tr\left(I_zM_0(\tau)I_z\right)/Tr\left(I_z^2\right).$$

The Eq. (21) is similar to Eq. (19), it can be transformed to local in time (Markov) form for small ω_1 by the same way, expanding upper limit of the integral to infinity and replacing $J_z(t-\tau)$ by $J_z(t)$. As a result for $t>T_2$ we obtain

$$\frac{\partial J_z(t)}{\partial t}=-R(\Delta)J_z(t),\quad R(\Delta)=\int_0^\infty dt\,W_0(t)=\pi\omega_1^2g(\Delta),\qquad g(\Delta)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}dte^{i\Delta t}F_0(t). \tag{22}$$

Direct application of the definition (2) produces famous relation for the free induction decay within the AWK theory:

$$F_0(t)=\left\langle e^{i\varphi(t,0)}\right\rangle_n=\exp\left(-\frac{1}{2}\int_0^t d\tau_1 d\tau_2\left\langle\omega_l(\tau_1)\omega_l(\tau_2)\right\rangle_n\right)=\exp\left(-M_2\int_0^t d\tau(t-\tau)\kappa(\tau)\right). \tag{23}$$

First condition of applicability of the Eq. (22) can be received again by retaining in (21) the term, linear in τ

$$\frac{\partial}{\partial t}J_z=-\omega_1^2\int_0^\infty d\tau\cos(\Delta\tau)F_0(\tau)\left(J_z(t)-\tau\frac{\partial}{\partial t}J_z(t)\right)=-R(\Delta)J_z+\vartheta\frac{\partial}{\partial t}J_z, \tag{24}$$

$$\vartheta=\omega_1^2\int_0^\infty dt\cdot t\cos(\Delta t)F_0(t)\le\omega_1^2\int_0^\infty dt\cdot tF_0(t)\sim\omega_1^2T_2^2=R_0T_2. \tag{25}$$

Therefore new term is negligible if $\vartheta\ll 1$, and condition $\varepsilon_1\ll 1$ is sufficient, but the condition $\vartheta\le R(\Delta)T_2$ is not fulfilled here, contrary to Eqs. (17), because $R(\Delta\tau_c\to\infty)$ decays exponentially (as a Fourier transform of a smooth function), while $\vartheta(\Delta\to\infty)\sim\Delta^{-2}$.

To obtain second condition of the applicability of perturbation theory we should calculate next term $M_1(t)$ of the expansion of the memory kernel (19) in powers of ω_1. It is of the form

$$M_1(t)=-\int_0^t dsduPL_1U_0(t,s)\overline{P}L_1U_0(s,u)\overline{P}L_1U_0(u,0)\overline{P}L_1P,\quad U_0(t,s)=\exp\left(-i\int_s^t d\tau L_0(\tau)\right). \tag{26}$$

This term produces correction $W_1(t)$ to the memory function $W_0(t)$ in the Eq. (21)

$$W_1(t)=\mathrm{Tr}\left(I_zM_1(t)I_z\right)/Tr\left(I_z^2\right)=-\omega_1^4\int_0^t ds\int_0^s du\left(S_1(t-s,s-u,u)-S_0(t-s)S_0(u)\right), \tag{27}$$

where $S_0(t-s)=\left\langle\mathrm{Re}\,e^{i\varphi(t,s)}\right\rangle$ and $S_1(t-s,s-u,u)=\left\langle\mathrm{Re}\,e^{i\varphi(t,s)}\,\mathrm{Re}\,e^{i\varphi(u,0)}\right\rangle$, together with correction of the saturation rate R in the Eq. (22):

$$R\to R+R_1,\qquad R_1=\int_0^\infty dt\;W_1(t). \tag{28}$$

Here and below the case $\Delta=0$ is discussed only. After transformations we obtain

$$R_1=R_1^{(+)}+R_1^{(-)},\qquad R_1^{(\pm)}=-\frac{\omega_1^4}{4}\int_0^\infty dtdsdue^{-Q(t)-Q(u)}\left(e^{\pm\Psi(t,s,u)}-1\right), \tag{29}$$

$$Q(t)=\frac{1}{2}M_2\int_0^t dudv\;\kappa(u-v)=M_2\int_0^t du(t-u)\kappa(u), \tag{30}$$

$$\Psi(t,s,u)=M_2\int_0^t dt'\int_0^u du'\kappa(t'+s+u'). \tag{31}$$

For preliminary qualitative understanding we should recognize, that essential range on t and u of the integrand (29) is of order T_2, while its duration on s has the order τ_c, because with increasing of s we have $S_1(t,s\to\infty,u)\to S_0(t)S_0(u)$ (or $\Psi(t,s\to\infty,u)\to 0$) that produces a rough estimation $R_1\sim\omega_1^4T_2^2\tau_c$.

If $\tau_c\ll T_2$, then this estimation is sufficient, but opposite relation $\tau_c\gg T_2\sim M_2^{-1/2}$ requires more detailed analysis. Below, following to Refs. [5] and [3], we apply rather general form of the local field correlation function

$$\kappa(t)=\left(\left(T_{2T}+\tau_0\right)/\left(\left(t^2+T_{2T}^2\right)^{1/2}+\tau_0\right)\right)^{3/2}. \tag{32}$$

This relation includes all existing qualitative information about correlation function: existence of smooth quadratic in time evolution at $t<T_{2T}$, its transformation into linear in time dependence at $T_{2T}<t<\tau_0$ with consequent transformation to 3d-diffusional asymptotics $\kappa(t)\sim t^{-3/2}$ at $t\gg\tau_0$. It is evident, that $\tau_c\sim T_{2T}+\tau_0$ here.

Substituting this correlation function in (29) – (31) we obtain second condition of applicability of the Eq. (22)

$$\varepsilon_2=R_1/R_0\sim R_0\tau_c\left(1+\left(T_{2T}\tau_0\right)/\tau_c^2\ln^2\left(T_{2T}^3/\left(T_2^2\tau_0\right)\right)\right)^{1/2}\ll 1, \tag{33}$$

that for $T_{2T}\sim\tau_0\sim\tau_c$ is equivalent to

$$\varepsilon_2=R_1/R_0\sim R_0\tau_c\left(1+\ln^2\left(\tau_c/T_2\right)\right)^{1/2}\ll 1. \tag{34}$$

Similar condition was derived in Ref. [5] for two-spin cross-relaxation transitions, that is natural, because two-spin cross-relaxation problem can be reduced to one-spin evolution with the Hamiltonian (1), see for example [15].

3. Very slow evolution of local fields

If the local field evolves very slowly, then according to the adiabatic theorem of Landau-Majorana-Stückelberg-Zener [16] the projection $\mathbf{J}(t)\mathbf{\Omega}(\mathrm{t})/\Omega(\mathrm{t})$ of the spin $\mathbf{J}(t)$ on the effective field $\mathbf{\Omega}(t)=\left(\omega_1,0,\omega_l(t)\right)$ is adiabatic invariant. The case $\Delta=0$ is discussed below only. We can suppose that $\tau_c\gg T_2\approx\left(\pi/\left(2M_2\right)\right)^{1/2}\ll\omega_1^{-1}$ and introduce the time of averaging T_{av} for which $\tau_c\gg T_{av}\gg T_2$. The evolution starts from initial state ρ_0 (4) and after the short time T_{av} $\bar{J}_z(t=T_{av})=\bar{J}_z^0\approx p_0$. Here and below upper bar indicates averaging during the time T_{av}. Later we will omit the difference $\left(p_0-\bar{J}_z^0\right)/p_0\sim\omega_1T_2\ll 1$ and use $\bar{J}_z^0=p_0$. In most of time $\left|\omega_l(t)\right|\gg\omega_1$, and, as a result of the adiabatic theorem, at this time $\bar{J}_z(t)=\pm p_0$ corresponding to the sign of $\omega_l(t)$. Therefore in these conditions

$$F(t)=\frac{\left\langle I_z(t)I_z\right\rangle}{\left\langle I_z^2\right\rangle}=\left\langle sign(\omega_l(t))\cdot sign(\omega_l(0))\right\rangle_n. \tag{35}$$

Further calculation should be based on main definition (2). Using the Fourier-transformation, we have

$$sign(x)=\int_{-\infty}^{\infty}\frac{dq}{2\pi}\cdot e^{iqx}s(q),\quad s(q)=\int_{-\infty}^{\infty}dxe^{-iqx-\eta|x|}sign(x)=-\frac{2iq}{\eta^2+q^2},\quad \eta\to+0. \tag{36}$$

Substituting this Fourier decomposition into (35), we have

$$F(t)=\int_{-\infty}^{\infty}\frac{dq_1dq_2}{\left(2\pi\right)^2}s^*(q_1)s(q_2)\left\langle e^{-iq_1\omega_l(t)+iq_2\omega_l(0)}\right\rangle_n=$$

$$= \int_{-\infty}^{\infty} \frac{dq_1 dq_2}{(2\pi)^2} s^*(q_1) s(q_2) \exp\left[-\frac{M_2}{2}\left(q_1^2 + q_2^2 - 2q_1 q_2 \kappa(t)\right)\right]. \tag{37}$$

It is evident from (36) and (37) that

$$\frac{d}{d\kappa(t)} F(t) = 4M_2 \int_{-\infty}^{\infty} \frac{dq_1 dq_2}{(2\pi)^2} \exp\left[-\frac{M_2}{2}\left(q_1^2 + q_2^2 - 2q_1 q_2 \kappa(t)\right)\right] = \frac{2}{\pi\left(1-\kappa^2(t)\right)^{1/2}}. \tag{38}$$

Integration of the last equation produces final result

$$F(t) = 1 + \frac{2}{\pi}\int_1^{\kappa(t)} \frac{dy}{\left(1-y^2\right)^{1/2}} = \frac{2}{\pi}\arcsin\kappa(t). \tag{39}$$

Estimation of corrections to the relation (39) and elucidation of its range of applicability requires much more complex calculations, than the transformation from (35) to (39). It is natural as well as receiving of main order master equation (22) was much simpler, then derivation of the conditions of its applicability (33) and (34).

Standard condition of applicability of the adiabatic theorem on one trajectory requires

$$\varepsilon_{ad1} = \max\left|\frac{d}{dt}\mathbf{n}(t)\right| / \omega_1 \ll 1, \quad \mathbf{n}(t) = \frac{\mathbf{\Omega}(t)}{\Omega(t)}. \tag{40}$$

Direct calculation produces

$$\varepsilon_{ad1} = \frac{1}{\omega_1^2}\left|\frac{d\omega_l}{dt}\right|, \tag{41}$$

and this value is realized at $\omega_l = 0$. According to the adiabatic theorem [16] decrease of the polarization during one passage of the range $\omega_l \sim \omega_1$ is $\tilde{\delta}_{nad} = \exp\left(-\pi / \varepsilon_{ad1}\right) \ll 1$, that produces

$$\delta_{nad} = \left\langle \tilde{\delta}_{nad} \right\rangle_n = \left\langle \exp\left(-\pi/\varepsilon_{ad1}\right)\right\rangle_n \sim \exp\left(-\left(\frac{27\pi}{4}\omega_1^4 \tau'^2 T_2^2\right)^{1/3}\right), \tag{42}$$

where $T_2 = \left(\pi / \left(2M_2\right)\right)^{1/2}$ and $\tau' = \left|\partial^2 \kappa(t=0)/\partial t^2\right|^{-1/2} \sim \tau_c$. It was taken into account here that distribution of $d\omega_l / dt$ is Gaussian with $\left\langle \left(d\omega_l / dt\right)^2 \right\rangle_n = -M_2 d^2\kappa(t=0)/dt^2$. The relation (42) indicates, that the condition of applicability of the results (39) and (42) receives the form $\delta_{nad} \ll 1$ or $\left(\left(27\pi/4\right)\omega_1^4 \tau'^2 T_2^2\right)^{-1/3} \ll 1$.

Frequency of passages of the local field $\omega_l(t)$ near the value $\omega_l = 0$ is $W_{nad} \sim 1/\tau_c$, therefore nonadiabatic losses of polarization should follow the law $\exp\left(-\delta_{nad} W_{nad} t\right)$ for $\delta_{nad} W_{nad} t \lesssim 1$ at least. Accounting it, we get

$$F(t) = \frac{2}{\pi}\arcsin\kappa(t)\cdot\exp\left(-\delta_{nad} W_{nad} t\right). \tag{43}$$

The spin-lattice relaxation should be considered separately of course.

We expect, that the relation (43) can be useful in studies of quasi-Ising spin systems by the magnetic resonance of impurity spins and in quantum information processing, therefore additional theoretical and experimental studies are necessary.

4. Conclusions

The model (1) is very important in spin dynamics and physical kinetics. Content of the lecture bridges existing gaps in known textbooks and, we hope, will give new possibility for applications of the magnetic resonance.

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