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[*] In Kazan University the Electron Paramagnetic Resonance (EPR) was discovered by Zavoisky E.K. in 1944.

Magnetic properties and spin kinetics in Kondo lattices

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We present a theoretical model to describe unusual properties of Kondo lattices. The influence of the Kondo effect on the static magnetic susceptibility and electron spin resonance (ESR) parameters is studied in a simple molecular field approximation together with a scaling perturbative approach. Theoretical expressions well agree with the ESR and static magnetic susceptibility experimental data.

1. Introduction

The unexpected observation of the electron spin resonance (ESR) in the heavy fermion compounds YbRh$_2$Si$_2$ and YbIr$_2$Si$_2$ [1, 2] below the thermodynamically measured Kondo temperature T_K [3, 4] ($T_K \approx 25$ K for YbRh$_2$Si$_2$ and $T_K \approx 40$ K for YbIr$_2$Si$_2$) stimulated a renewed interest in the theory of Kondo lattices. A series of theoretical approaches was proposed to understand the ESR signal existence from both a Fermi-liquid description [5, 6] and a picture of localized Kondo-ions moments [7-20]. The latter approach is supported by the ESR data on the resonance g-factor, linewidth and intensity which certainly reflect the tetragonal symmetry of the crystal electric field (CEF) at the Yb^{3+} ion position [1-2, 21-24]. The static magnetic susceptibility measurement also points out localized nature of f-electron motion in the CEF [11].

In a series of previous works [11, 12, 14-20] we investigated the static properties and spin kinetics in Kondo lattices at temperatures below T_K basing on the entirely local properties of Yb^{3+} ions in the CEF. The theoretical expressions for the static magnetic susceptibility obtained in a simple molecular field approximation [11] give an excellent agreement with experimental data. In other works [12, 16-18] it was shown that the main reason of the ESR signal observability is the mutual cancelation of the large relaxation rates in the collective spin mode due to the Kondo effect. In this paper we give a brief review of the developed theory and present some new results on the ESR intensity and static magnetic susceptibility.

2. Basic Hamiltonian

We start from the atomic properties of an Yb^{3+} ion. A free Yb^{3+} ion has a 4f^{13} configuration with a single term ^{2}F. The spin orbital interaction splits the ^{2}F term into two multiplets: ^{2}F$_{7/2}$ with $J = 7/2$ and ^{2}F$_{5/2}$ with $J = 5/2$, where J denotes the value of the total momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ with $\mathbf{L}$ and $\mathbf{S}$ as the orbital and spin momentum of the ion. Since the excited multiplet ^{2}F$_{5/2}$ is separated from the ground one ^{2}F$_{7/2}$ by the energy interval 1 eV, which is very large compared with the CEF energy, we consider in the following the ground multiplet only. Within the lowest multiplet the spin and orbital momentums of the ion are expressed via its total electronic momentum: $\mathbf{S} = (g_J - 1)\mathbf{J}$, $\mathbf{L} = (2 - g_J)\mathbf{J}$, where $g_J = 8/7$ is the Lande g-factor for the ^{2}F$_{7/2}$ multiplet.

The basic model includes the Zeeman energy, the Kondo interaction of Yb^{3+} ions with conduction electrons, the coupling between Yb^{3+} ions via conduction electrons and the kinetic energy of conduction electrons. The energy of Yb^{3+} ions in external magnetic field $\mathbf{H}$ can be written as follows

$$H_{\text{Zeeman}} = \mu_B g_J \sum_i \mathbf{H} \mathbf{J}_i \,. \tag{1}$$

The Kondo exchange coupling of Yb^{3+} ions with conduction electrons and the indirect interaction between Yb^{3+} ions via conduction electrons (RKKY interaction) are also expressed in term of the total momentum $\mathbf{J} = (J^x, J^y, J^z)$:

$$H_{\text{Kondo}} = A_0(g_J - 1)\sum_i \mathbf{J}_i \boldsymbol{\sigma}_i, \tag{2}$$

$$H_{\text{RKKY}} = (g_J - 1)^2 \sum_{ij} I_{\text{RKKY}}^{ij} \mathbf{J}_i \mathbf{J}_j. \tag{3}$$

Here A_0 denotes the zero order term of the Kondo exchange integral expansion in multipoles [25-27], $\boldsymbol{\sigma}_i$ is the spin density of the conduction electrons at the i-th ion site and I_{RKKY}^{ij} denotes the constant of the RKKY interaction between two Kondo ions.

The tetragonal CEF splits the ground multiplet $^2F_{7/2}$ into four Kramers doublets, each one described by the wave functions of the type $\psi_\pm = \sum_M c_{\pm M} |\pm M\rangle$ [11, 14, 15], where M is the eigenvalue of operator J^z (z-axis coincides with tetragonal axis of CEF). Within every Kramers doublet the total electronic momentum of the Yb^{3+} ion can be represented by the effective spin $S = 1/2$:

$$J^z = \gamma_\| S^z, \qquad J^{x,y} = \gamma_\perp S^{x,y}, \tag{4}$$

where $\mathbf{S}$ is the effective spin operator, $\gamma_\|$ and $\gamma_\perp$ are given by

$$\gamma_\| = 2\langle \psi_+ | J^z | \psi_+ \rangle, \qquad \gamma_\perp = 2\langle \psi_+ | J^x | \psi_- \rangle. \tag{5}$$

Since the first excited level is separated from the ground one by 17 meV (197 K) [28] and 18 meV (209 K) [29] in the cases of YbRh$_2$Si$_2$ and YbIr$_2$Si$_2$ the low temperature physics ($T \ll 200\,\text{K}$) are well described by the lowest Kramers doublet. After projection onto the ground state the Zeeman energy, the Kondo interaction and the RKKY interaction take the form

$$H_{Zs} = \mu_B \sum_i \left[g_\perp^s (H^x S_i^x + H^y S_i^y) + g_\|^s H^z S_i^z \right], \qquad g_{\perp,\|}^s = g_J \gamma_{\perp,\|}, \tag{6}$$

$$H_{s\sigma} = \sum_i \left[J_\perp (S_i^x \sigma_i^x + S_i^y \sigma_i^y) + J_\| S_i^z \sigma_i^z \right], \qquad J_{\perp,\|} = J g_{\perp,\|}^s, \quad J = A_0(g_J - 1)/g_J, \tag{7}$$

$$H_{ss} = \sum_{ij} \left[I_\perp^{ij} (S_i^x S_j^x + S_i^y S_j^y) + I_\|^{ij} S_i^z S_j^z \right], \qquad I_{\perp,\|}^{ij} = (g_{\perp,\|}^s)^2 I^{ij}, \quad I^{ij} = I_{\text{RKKY}}^{ij}(g_J - 1)^2 / g_J^2. \tag{8}$$

The anisotropies of the Kondo and RKKY interactions are evidently related to that of the g-factor:

$$J_\perp / J_\| = g_\perp^s / g_\|^s, \qquad I_\perp^{ij} / I_\|^{ij} = \left(g_\perp^s / g_\|^s \right)^2. \tag{9}$$

Note, that the coordinate axes x, y, z in expressions (4)-(8) coincide with the crystallographic axes a, b, c.

The kinetic energy of conduction electrons and their Zeeman energy can be written as

$$H_c = \sum_{ij\lambda} t_{ij} c_{i\lambda}^+ c_{j\lambda} - \mu \sum_{i\lambda} c_{i\lambda}^+ c_{i\lambda}, \tag{10}$$

$$H_{Z\sigma} = \mu_B g^\sigma \sum_i \mathbf{H}\boldsymbol{\sigma}_i. \tag{11}$$

Here $\lambda = \pm 1$ labels the orientation of the conduction electrons spin, μ is the chemical potential, g^σ denotes the g-factor of the conduction electrons. The conduction electrons density is expressed in terms of the creation and annihilation operators

$$\boldsymbol{\sigma}_i = \sum_{\lambda\lambda'} \mathbf{s}_{\lambda\lambda'}\, c_{i\lambda}^+ c_{i\lambda'}, \tag{12}$$

where $\mathbf{s}_{\lambda\lambda'}$ are the matrix elements of spin operators $s = 1/2$.

Finally, we represent the total Hamiltonian as $H = H_0 + H_{\text{int}}$ with

$$H_0 = H_c + H_{Zs} + H_{Z\sigma}, \qquad H_{\text{int}} = H_{s\sigma} + H_{ss}. \tag{13}$$

3. Static magnetic susceptibility

At first we consider the effects of the Kondo and RKKY interactions on the static magnetic susceptibility and *g*-factors in a simple molecular field approximation. In this case the Zeeman energy of the Yb^{3+} ions and conduction electrons is renormalized and the total Hamiltonian (13) is reduced to

$$\tilde{H} = H_c + \tilde{H}_{Zs} + \tilde{H}_{Z\sigma} \tag{14}$$

with

$$\tilde{H}_{Zs} = \mu_B \sum_i \mathbf{H} \hat{\tilde{g}}^s \mathbf{S}_i, \qquad \tilde{H}_{Z\sigma} = \mu_B \sum_i \mathbf{H} \hat{\tilde{g}}^\sigma \boldsymbol{\sigma}_i. \tag{15}$$

The renormalized *g*-tensors $\hat{\tilde{g}}^s$ and $\hat{\tilde{g}}^\sigma$ are expressed via the static magnetic susceptibility by the relations

$$\hat{\tilde{g}}^s = \left(1 - \lambda_J \hat{\chi}^\sigma - \lambda_I \hat{\chi}^s\right)\hat{g}^s, \qquad \hat{\tilde{g}}^\sigma = \left(1 - \lambda_J \hat{\chi}^s\right) g^\sigma, \tag{16}$$

where

$$\lambda_J = \frac{J}{N\mu_B^2 g^\sigma}, \qquad \lambda_I = \frac{1}{N\mu_B^2}\sum_i I^{ij}, \tag{17}$$

J and I^{ij} are introduced in (7) and (8), N is the number of Yb^{3+} ions. The static magnetic susceptibility tensors are defined in a usual way:

$$\chi_{\alpha\beta}^s = \left.\frac{\partial \langle M_\alpha^s \rangle}{\partial H^\beta}\right|_{\mathbf{H}=0}, \qquad \chi_{\alpha\beta}^\sigma = \left.\frac{\partial \langle M_\alpha^\sigma \rangle}{\partial H^\beta}\right|_{\mathbf{H}=0}, \qquad \mathbf{M}^s = -\mu_B \hat{g}^s \sum_i \mathbf{S}_i, \qquad \mathbf{M}^\sigma = -\mu_B g^\sigma \sum_i \boldsymbol{\sigma}_i, \tag{18}$$

where $\langle \dots \rangle$ means the thermodynamical average and $\alpha, \beta = x, y, z$.

In the molecular field approximation the conduction electrons and Yb^{3+} ions susceptibilities follow a system of self-consistent equations

$$\begin{cases} \left(1 + \lambda_I \hat{\chi}^{0s}\right)\hat{\tilde{\chi}}^s + \lambda_J \hat{\chi}^{0s}\hat{\tilde{\chi}}^\sigma = \hat{\chi}^{0s}, \\ \lambda_J \hat{\chi}^{0\sigma}\hat{\tilde{\chi}}^s + \hat{\tilde{\chi}}^\sigma = \hat{\chi}^{0\sigma}. \end{cases} \tag{19}$$

Here $\hat{\chi}^{0s}$ and $\hat{\chi}^{0\sigma}$ are the static susceptibilities unaffected by the Kondo and RKKY interactions. The elementary calculations with the Hamiltonian H_0 leads to the relations $\chi_{\alpha\beta}^{0s} = \delta_{\alpha\beta}\chi_{\alpha\alpha}^{0s}$ and $\chi_{\alpha\beta}^{0\sigma} = \delta_{\alpha\beta}\chi^{0\sigma}$. The non-zero components $\chi_{xx}^{0s} = \chi_{yy}^{0s} = \chi_\perp^{0s}$ and $\chi_{zz}^{0s} = \chi_\parallel^{0s}$ are the well-known Curie susceptibility and $\chi^{0\sigma}$ is the usual Pauli susceptibility:

$$\chi_{\perp,\parallel}^{0s} = \frac{C_{\perp,\parallel}^0}{T}, \qquad C_{\perp,\parallel}^0 = \frac{N(\mu_B g_{\perp,\parallel}^s)^2}{4k_B}, \tag{20}$$

$$\chi^{0\sigma} = \frac{1}{2}N(\mu_B g^\sigma)^2 \rho, \tag{21}$$

where ρ denotes the conduction electrons density of states per lattice site at the Fermi surface.

The symmetry of $\hat{\chi}^{0s}$ and $\hat{\chi}^{0\sigma}$ tensors implies the relations $\tilde{\chi}_{\alpha\beta}^s = \delta_{\alpha\beta}\tilde{\chi}_{\alpha\alpha}^s$ and $\tilde{\chi}_{\alpha\beta}^{\sigma} = \delta_{\alpha\beta}\tilde{\chi}_{\alpha\alpha}^{\sigma}$ for the solution of the system (19). The non-zero components of the Yb^{3+} ions susceptibility $\tilde{\chi}_{xx}^s = \tilde{\chi}_{yy}^s = \tilde{\chi}_{\perp}^s$ and $\tilde{\chi}_{zz}^s = \tilde{\chi}_{\parallel}^s$ follow the Curie-Weiss law

$$\tilde{\chi}_{\perp,\parallel}^s = \frac{C_{\perp,\parallel}}{T + \theta_{\perp,\parallel}} \tag{22}$$

with the Curie constant and Weiss temperature

$$C_{\perp,\parallel} = C_{\perp,\parallel}^0 \left(1 - \lambda_J \chi^{0\sigma}\right), \qquad \theta_{\perp,\parallel} = C_{\perp,\parallel}^0 \left(\lambda_I - \lambda_J^2 \chi^{0\sigma}\right). \tag{23}$$

The conduction electrons susceptibility becomes anisotropic and temperature dependent as affected by the Kondo and RKKY interactions:

$$\tilde{\chi}_{\perp,\parallel}^{\sigma} = \chi^{0\sigma}\left(1 - \lambda_J \frac{C_{\perp,\parallel}}{T + \theta_{\perp,\parallel}}\right). \tag{24}$$

It is evident that the main contribution to the total susceptibility $\chi = \chi^s + \chi^{\sigma}$ is given by the Yb^{3+} ions susceptibility due to the small value of the ratio $\chi^{\sigma}/\chi^s \sim \rho T$.

Substituting (22)-(24) into (16) we obtain the *g*-factors of Yb^{3+} ions and conduction electrons which are also anisotropic and temperature dependent:

$$\tilde{g}_{\perp,\parallel}^s = g_{\perp,\parallel}^s\left(1 - \lambda_J \chi^{0\sigma}\right)\frac{T}{T + \theta_{\perp,\parallel}}, \qquad \tilde{g}_{\perp,\parallel}^{\sigma} = g^{\sigma}\left(1 - \lambda_J \frac{C_{\perp,\parallel}}{T + \theta_{\perp,\parallel}}\right). \tag{25}$$

In previous works [11, 19, 20] we studied the static magnetic susceptibility taking into account the excited states of the CEF as well, which results in an additional temperature independent Van Vleck term. The Curie-Weiss susceptibility together with Van Vleck part is well fitted to experimental data on the static susceptibility [11]. On the other hand the measurements of the ESR intensity [22], which is proportional to the static susceptibility, show the Curie-Weiss law alone. If we are interested in the ESR study it is sufficient to consider the Zeeman-splitted ground doublet in resonance with the alternating magnetic field. The Curie-Weiss term can be interpreted as a resonant part of the total static susceptibility. Although the molecular field approximation provides a good agreement with experiment, the fitting parameters C and θ turned to be different for low and high temperature [11]. This temperature dependence may indicate the Kondo effect which leads to an additional renormalization of the static susceptibility.

A perturbation theory might seem a first step to go beyond the molecular field approximation. However, the standard perturbation expansion in the Kondo interaction is not sufficient at low temperatures: the second order calculations reveal the logarithmic divergencies of the type $\ln(T/W)$, where W is a conduction electron bandwidth. This trouble can be overcome by means of the Anderson's "poor man's scaling" method [30] which allows one to extend the lowest order perturbation result and effectively sum the leading order logarithmic terms. The idea of this approach is to take into account the effect of the high energy excitations on the low energy physics by a renormalization of coupling constants. The original Kondo interaction $H_{s\sigma}$ (7) is projected onto the low energy conduction electrons states yielding a Hamiltonian $H_{s\sigma}'$ with new Kondo couplings $J_{\perp}'$ and $J_{\parallel}'$ [16-20]. The renormalized dimensionless parameters $U_{\perp} = (\rho J_{\perp})'$ and $U_{\parallel} = (\rho J_{\parallel})'$ become temperature dependent:

$$U_{\perp} = \bar{U}/\sin\varphi, \qquad U_{\parallel} = \bar{U}\cot\varphi, \qquad \varphi = \bar{U}\ln(T/T_{GK}), \qquad \bar{U} = \rho\sqrt{J_{\perp}^2 - J_{\parallel}^2}, \tag{26}$$

where T_{GK} denotes a characteristic temperature given as follows

$$T_{GK} = W \exp\left[-\frac{1}{\bar{U}} \arccos\left(g_\parallel^s \big/ g_\perp^s \right) \right] \tag{27}$$

(the abbreviation "GK" indicates the Kramers ground state). The quantities T_{GK} and $\bar{U}$ are scaling invariants which do not change with renormalizing the Hamiltonian $H_{s\sigma}$.

Using the "poor man's scaling" method we find the static magnetic susceptibilities and the g-factors renormalized by the high energy conduction electrons excitations:

$$(\chi_{\perp,\parallel}^s)' = \frac{C_{\perp,\parallel}'}{T + \theta_{\perp,\parallel}'}, \qquad (\chi_{\perp,\parallel}^\sigma)' = \chi^{0\sigma}\left(1 - \lambda_J \frac{C_{\perp,\parallel}'}{T + \theta_{\perp,\parallel}'} \right) \tag{28}$$

and

$$(g_{\perp,\parallel}^s)' = g_{\perp,\parallel}^s \left(1 - \lambda_J \chi^{0\sigma} \right) \frac{T}{T + \theta_{\perp,\parallel}'}, \qquad (g_{\perp,\parallel}^\sigma)' = g^\sigma \left(1 - \lambda_J \frac{C_{\perp,\parallel}'}{T + \theta_{\perp,\parallel}'} \right), \tag{29}$$

where

$$C_{\perp,\parallel}' = C_{\perp,\parallel} Z_{\perp,\parallel}, \qquad \theta_{\perp,\parallel}' = \theta_{\perp,\parallel} Z_{\perp,\parallel} \tag{30}$$

with

$$Z_\perp = 1 + \bar{U}\left(1/\sin\varphi_0 - 1/\sin\varphi \right), \qquad Z_\parallel = 1 + \bar{U}\left(\cot\varphi_0 - \cot\varphi \right). \tag{31}$$

Here $\varphi_0 = \varphi(T = W) = \arccos\left(g_\parallel^s \big/ g_\perp^s \right)$; $\bar{U}$ and φ are introduced in (26).

Formally, the expressions (28) and (29) are the same as that obtained in the molecular field approximation but the Curie constants and Weiss temperature convert to temperature dependent functions. The new Curie and Weiss parameters decrease upon lowering temperature and their asymptotic behavior at $T \ll T_{GK} \exp(1/\bar{U})$ follows

$$C_{\perp,\parallel}' = C_{\perp,\parallel}\left[1 + \rho J_{\perp,\parallel} - \frac{1}{\ln(T/T_{GK})} \right], \qquad \theta_{\perp,\parallel}' = \theta_{\perp,\parallel}\left[1 + \rho J_{\perp,\parallel} - \frac{1}{\ln(T/T_{GK})} \right]. \tag{32}$$

Such a reduction of the Curie and Weiss "constants" at low temperature is in a qualitative agreement with experimental data [11].

4. The ESR parameters

To study ESR we add to the Hamiltonian (13) the interaction of conduction electrons and localized moments with an external alternating magnetic field perpendicular to the static magnetic field:

$$H_{mw} = \mu_B g^\sigma \sum_i \mathbf{h}_{mw} \boldsymbol{\sigma}_i + \mu_B \sum_i \left[g_\perp^s \left(h_{mw}^x S_i^x + h_{mw}^y S_i^y \right) + g_\parallel^s h_{mw}^z S_i^z \right], \tag{33}$$

where $\mathbf{h}_{mw} = \mathbf{h}_0 \cos\omega t$; $\mathbf{h}_0$ and ω are the amplitude and frequency of the microwave field, respectively. The ESR response due to the microwave magnetic field perturbation (33) is given by the transverse dynamical susceptibility

$$\chi(\omega) = -\left\langle\left\langle M_h \big| M_h \right\rangle\right\rangle, \tag{34}$$

where $M_h = M_h^s + M_h^\sigma$ is the total magnetic moment along the direction of the microwave field, $\left\langle\left\langle A \big| B \right\rangle\right\rangle$ means the Fourier transform of a retarded Green function

$$\left\langle\left\langle A\,|\,B\right\rangle\right\rangle = -i\int\limits_0^\infty dt\,\left\langle\left[A(t),B\right]\right\rangle\exp(i\omega t). \tag{35}$$

For the sake of simplicity we consider the case of the static magnetic field parallel to the crystal symmetry axis c (z), although the calculations are principally the same for an arbitrary orientation of the static and alternating fields to the crystallographic axes. In this case

$$\chi(\omega) = \sum_{\alpha\beta}\chi_{\alpha\beta}(\omega), \qquad \alpha,\beta = s,\sigma \tag{36}$$

with partial susceptibilities $\chi_{\alpha\beta}(\omega)$:

$$
\begin{aligned}
\chi_{ss} &= -(\mu_B g_\perp^s)^2\left\langle\left\langle S^-\,\big|\,S^+\right\rangle\right\rangle, & \chi_{s\sigma} &= -\mu_B^2 g_\perp^s g^\sigma\left\langle\left\langle S^-\,\big|\,\sigma^+\right\rangle\right\rangle, \\
\chi_{\sigma s} &= -\mu_B^2 g_\perp^s g^\sigma\left\langle\left\langle \sigma^-\,\big|\,S^+\right\rangle\right\rangle, & \chi_{\sigma\sigma} &= -(\mu_B g^\sigma)^2\left\langle\left\langle \sigma^-\,\big|\,\sigma^+\right\rangle\right\rangle.
\end{aligned}
\tag{37}
$$

Here $\mathbf{S}$ and $\boldsymbol{\sigma}$ are the spin operators of Yb^{3+} ions and conduction electrons, respectively; $S^\pm = (S^x \pm iS^y)/\sqrt{2}$, $\sigma^\pm = (\sigma^x \pm i\sigma^y)/\sqrt{2}$.

The collective spin motion of conduction electrons and localized moments is described by a set of coupled equations

$$
\begin{pmatrix} a_{ss} & a_{s\sigma} \\ a_{\sigma s} & a_{\sigma\sigma} \end{pmatrix}
\begin{pmatrix} \chi_{ss} & \chi_{s\sigma} \\ \chi_{\sigma s} & \chi_{\sigma\sigma} \end{pmatrix}
=
\begin{pmatrix} P_{ss} & 0 \\ 0 & P_{\sigma\sigma} \end{pmatrix}
\tag{38}
$$

with

$$
a_{\alpha\alpha} = \omega - \omega_\alpha + \Sigma_{\alpha\alpha} + \Sigma_{\alpha L}\ (\alpha = s,\sigma), \qquad
a_{s\sigma} = \lambda_J\chi_\perp^{0s}\omega_s - \frac{g_\perp^s}{g^\sigma}\Sigma_{s\sigma}, \qquad
a_{\sigma s} = \lambda_J\chi^{0\sigma}\omega_\sigma - \frac{g^\sigma}{g_\perp^s}\Sigma_{\sigma s},
$$
$$
P_{ss} = -\chi_\perp^{0s}\omega_s, \qquad P_{\sigma\sigma} = -\chi^{0\sigma}\omega_\sigma.
\tag{39}
$$

Here ω_s and ω_σ are the resonant frequencies of Yb^{3+} ions and conduction electrons, which include only the molecular field shifts due to the Kondo and RKKY interactions:

$$\omega_s = \mu_B\tilde{g}_\|^s H, \qquad \omega_\sigma = \mu_B\tilde{g}_\|^\sigma H; \tag{40}$$

λ_J, $\tilde{g}_\|^s$ and $\tilde{g}_\|^\sigma$ are defined in (17) and (25). The imaginary parts of kinetic coefficients $\Gamma_{\alpha\beta} = \mathrm{Im}\left(\Sigma_{\alpha\beta}\right)$ represent the partial relaxation rates in the system of Yb^{3+} ions and conduction electrons and their real parts give additional shifts to corresponding resonant frequencies.

The kinetic coefficients Γ_{ss} and $\Gamma_{\sigma\sigma}$ are the well-known Korringa and Overhauser relaxation rates (Yb^{3+} ions relax to the conduction electrons being in the thermodynamical equilibrium and vice versa). Two additional coefficients Γ_{sL} and $\Gamma_{\sigma L}$ describe the relaxation of Kondo ions and conduction electrons to the thermal bath ("lattice"). Finally, the coefficients $\Gamma_{s\sigma}$ and $\Gamma_{\sigma s}$ provide the coupling between the transvers magnetizations of Yb^{3+} and conduction electrons. This coupling is especially important if the relaxation rate of the conduction electrons toward the Kondo ions is much faster than to the lattice and the resonant frequencies are close to one another ("bottleneck" regime):

$$\Gamma_{\sigma\sigma} \gg \Gamma_{\sigma L}, \left|\omega_s - \omega_\sigma\right|. \tag{41}$$

The poles of the total susceptibility are determined by the condition $a_{ss}a_{\sigma\sigma} - a_{s\sigma}a_{\sigma s} = 0$ which leads to two complex roots. Their real parts represent resonant frequencies and their imaginary parts represent the relaxation rates. Under condition of the strong bottleneck regime (41) the imaginary parts of the resonant poles read as follows

$$\mathrm{Im}(\omega_1) = \Gamma_{\sigma\sigma}, \tag{42}$$

$$\mathrm{Im}(\omega_2) = \Gamma_{s\mathrm{L}} + \tilde{\Gamma}_{\sigma\mathrm{L}} + \tilde{\Gamma}_{ss}, \quad \tilde{\Gamma}_{\sigma\mathrm{L}} = \Gamma_{\sigma\mathrm{L}} \frac{\Gamma_{s\sigma}\Gamma_{\sigma s}}{\Gamma_{\sigma\sigma}^2}, \quad \tilde{\Gamma}_{ss} = \Gamma_{ss} - \frac{\Gamma_{s\sigma}\Gamma_{\sigma s}}{\Gamma_{\sigma\sigma}}. \tag{43}$$

The expressions (42) and (43) were investigated in detail in the works [12, 16-18]. In the presence of the Kondo effect the partial relaxation rates diverge upon lowering temperature to T_{GK}. To the leading order in logarithmic terms

$$\Gamma_{ss} = \Gamma_{\sigma s} = \frac{\pi T}{\ln^2\left(T/T_{\mathrm{GK}}\right)}, \quad \Gamma_{\sigma\sigma} = \Gamma_{s\sigma} = \frac{\pi}{2\rho\ln^2\left(T/T_{\mathrm{GK}}\right)}. \tag{44}$$

The imaginary parts of the resonant poles take the form

$$\mathrm{Im}(\omega_1) = \frac{\pi}{2\rho\ln^2\left(T/T_{\mathrm{GK}}\right)}, \tag{45}$$

$$\mathrm{Im}(\omega_2) = \Gamma_{s\mathrm{L}} + 2\rho T \Gamma_{\sigma\mathrm{L}} + \frac{\pi}{4} T \bar{U}^4 \ln^2\left(T/T_{\mathrm{GK}}\right). \tag{46}$$

One can see that the first pole corresponds to the individual subsystems motion with the ESR linewidth too large to be experimentally observable. On the contrary, the second pole describes the strongly coupled spin motion of conduction electrons and Yb^{3+} ions when the individual subsystems features are not important. In this case the effective relaxation rate $\Gamma_{\mathrm{coll}} = \mathrm{Im}(\omega_2)$ is greatly reduced and determined mainly by the relaxation of Kondo ions to the lattice. The divergent parts of different kinetic coefficients cancel each other in the collective spin mode due to the existence of the common energy scale T_{GK} regulating their temperature dependence at $T \to T_{\mathrm{GK}}$.

A similar cancelation of the logarithmic terms of the kinetic coefficients $\Sigma_{\alpha\beta}$ occurs for the resonant frequency determined as a real part of the pole ω_2. The molecular field shifts arising from the Kondo interaction also disappear from the collective spin mode. However, the Knight shift due to RKKY interaction still takes place and the resonant frequency contains the Kondo anomaly term via the renormalized Weiss temperature. For the corresponding resonant g-factor $g_\parallel^{\mathrm{res}} = \mathrm{Re}(\omega_2)/\mu_{\mathrm{B}}H_{\mathrm{res}}$ we obtain

$$g_\parallel^{\mathrm{res}} = g_\parallel^s \frac{T}{T + \theta_\parallel(1 + \rho J_\parallel - \bar{U}\cot\varphi)}, \tag{47}$$

where $\theta_\parallel$ and φ are introduced in (23) and (26). In the absence of the RKKY interaction the resonant g-factor coincides with the usual g-factor of an Yb^{3+} ion in the tetragonal CEF.

Now we consider the ESR intensity, which is determined by integrating the absorbed power of the microwave magnetic field $\mathbf{h}_{\mathrm{mw}}$. The latter is expressed via the transverse dynamical susceptibility by the relation

$$P = \frac{1}{2} h_0^2 \omega \, \mathrm{Im}\left[\chi(\omega + i0)\right]. \tag{48}$$

Among the partial susceptibilities $\chi_{\alpha\beta}(\omega)$ the main contribution to the total susceptibility is made by the term $\chi_{ss}(\omega)$ due to the negligibly small value of the ratio $\chi^\sigma/\chi^s \sim \rho T$. In this approximation the elementary integrating gives

$$I \sim \omega_{\mathrm{res}}^2 \, \chi_h^s, \tag{49}$$

where ω_{res} is the microwave field frequency and χ_h^s denotes the static magnetic susceptibility of Yb^{3+}

ions along the direction of the alternating field. Note, that the expression (49) is valid for arbitrary directions of the static and alternating magnetic fields (the condition $\mathbf{H} \perp \mathbf{h}$ is always fulfilled).

Since the microwave frequency is constant in the ESR experiment the temperature dependence of the ESR intensity is determined by the static susceptibility only. As a result we have the Curie-Weiss law with the temperature dependent Curie and Weiss parameters:

$$ I_{\perp,\parallel} \sim \frac{C'_{\perp,\parallel}}{T + \theta'_{\perp,\parallel}}, \tag{50} $$

where $C'_{\perp,\parallel}$ and $\theta'_{\perp,\parallel}$ are defined by the expressions (30)-(32). Such a dependence qualitatively agrees with experimental data [22].

5. Summary

We theoretically investigated magnetic properties and spin kinetics of heavy fermion Kondo lattices $YbRh_2Si_2$ and $YbIr_2Si_2$ at temperatures well below thermodynamically measured Kondo temperature T_K. It turned out that the local properties of f-electrons in the CEF and the Kondo effect due to the strong interaction of Yb^{3+} ions with conduction electrons can explain many features of Kondo lattices with heavy fermions. It was shown that the Kondo effect plays an important part in the behavior of the Kondo lattice systems. The Kondo anomalies are manifested in the static magnetic susceptibility, the ESR linewidth, the resonant *g*-factor and the ESR intensity. The temperature dependence of these parameters obtained in the framework of proposed model well agrees with experimental data.

Acknowledgments

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