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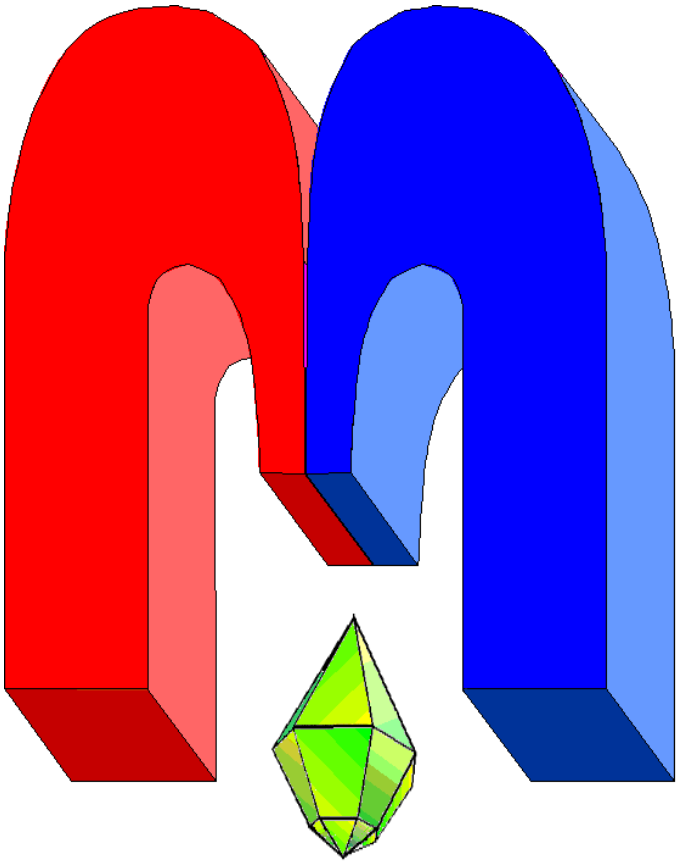

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* In Kazan University the Electron Paramagnetic Resonance (EPR) was discovered by Zavoisky E.K. in 1944.

Spin kinetics in Kondo lattice with heavy fermions

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We investigated the spin dynamics in the heavy fermion compounds $YbRh_2Si_2$ and $YbIr_2Si_2$. The contributions of the resonant and nonresonant parts of the total transverse magnetization to the electron spin resonance (ESR) parameters are analyzed for different orientations of the static and microwave magnetic fields. It is shown that at high temperatures, when the Kondo effect is absent, the nonresonant terms may be essential in the case of the perpendicular orientation of the static magnetic field to the crystal symmetry axis. In the presence of the Kondo effect the nonresonant parts do not make a significant contribution to the ESR parameters for any configuration of the magnetic fields.

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1. Introduction

The discovery of electron spin resonance (ESR) in the heavy fermion compound $YbRh_2Si_2$ [1] has led to considerable efforts, both theoretical and experimental, to understand Kondo lattice systems [2-10]. The ESR signal was observed well below the thermodynamically measured Kondo temperature $T_K \approx 25\,\mathrm{K}$ [11, 12], where the magnetic moments are supposed to be screened by conduction electrons. Moreover, the expected ESR linewidth is of the order $\Delta\nu = k_B T_K / 2\pi\hbar = 500\,\mathrm{GHz}$. The experimental results were completely opposite: at X-band (9.4 GHz) and $T = 0.7\,\mathrm{K}$ a linewidth of 0.3 GHz was observed. Similar results were obtained later for $YbIr_2Si_2$ [13, 14] ($T_K \approx 40\,\mathrm{K}$ [15]).

Abrahams and Wölfle [2, 3] studied the ESR in heavy fermion systems using a Fermi liquid description in the framework of the Anderson model, where the local magnetic moment is that of a quasi-localized f-electron. Schlottmann [4] gave an explanation of the ESR signal existence based on the Kondo lattice model with an isotropic interaction between the conduction electrons and the Kondo ions. However, both of these approaches do not take into account the strong spin orbital interaction and the crystal electric field (CEF) effects which result in the anisotropy of the Kondo interaction similar to that of the Zeeman energy. On the contrary a semiphenomenological theory presented by Huber [5-7] takes into account the anisotropy of the static and dynamical susceptibilities. The author was able to describe the ESR data in Yb-heavy fermion compounds, especially their angular dependence, but the analysis did not touch the reasons of the ESR signal observability assuming it a-priori.

In earlier works [16, 8-10] we proposed another approach to study static and dynamic properties of Kondo lattice systems. It is well known that unusual properties of heavy fermion compounds are determined by the interplay of the strong repulsion of 4f-electrons on the rare earth ion sites, their hybridization with wide band conduction d-electrons and the CEF effects. Recent angle resolved photoemission measurements [17] revealed the dispersion of the CEF-split 4f states due to f-d hybridization which was interpreted within the Anderson model. At the same time the rather narrow 4f band near the Fermi energy points out the quasi-localized nature of the f-electron motion. The ESR experiments also indicate the importance of local properties: the angular dependence of the *g*-factor and the ESR linewidth reflects the tetragonal symmetry of the CEF at the Yb-ion position. Starting from the entirely local properties of an Yb-ion in the CEF we investigated the static magnetic susceptibility of $YbRh_2Si_2$ and $YbIr_2Si_2$ at temperatures below T_K [16]. In other works [8-10] it was

shown that the collective spin motion of quasi-localized f-electrons and wide-band conduction d-electrons is the key ingredient for understanding the ESR signal existence in a Kondo lattice with heavy fermions. The strong coupling between the f- and d-electrons turned out to make a negligible contribution to the effective relaxation rate in the bottleneck regime. The ESR response is determined by the relaxation of the f- and d-electrons to the thermal bath rather than by their mutual relaxation. The peculiarities of the f-electrons band structure due to the f-d hybridization do not seem to be important for the study of the ESR phenomenon. Our model successfully explained the ESR data of $YbRh_2Si_2$ and $YbIr_2Si_2$ in terms of their dependencies on temperature and the orientations of the static and microwave magnetic fields. In this paper we present more accurate analysis of spin dynamics in Kondo lattice systems which takes into account all parts of the total dynamical susceptibility including the terms dropped previously as nonresonant ones.

2. Basic model

Our basic theoretical model includes the kinetic energy of conduction electrons, the Zeeman energy, the Kondo interaction between Yb-ions and conduction electrons and the coupling between Yb-ions via conduction electrons (RKKY interaction).

We start from the local properties of an Yb-ion in the crystal electric field. A free Yb^{3+}-ion has a $4f^{13}$ configuration with one term ^2F. The spin orbital interaction splits the ^2F term into two multiplets: $^2F_{7/2}$ with $J = 7/2$ and $^2F_{5/2}$ with $J = 5/2$, where J denotes the value of the total momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ with $\mathbf{L}$ and $\mathbf{S}$ as the orbital and spin momentum of the ion. The excited multiplet $^2F_{5/2}$ is separated from the ground state $^2F_{7/2}$ by about 1 eV. Since this value is much larger than the CEF energy, we consider in the following the ground multiplet only.

Within the lowest multiplet the spin and orbital momentums of the ion are expressed via its total electronic momentum and the Lande g-factor g_J: $\mathbf{S} = (g_J - 1)\mathbf{J}$, $\mathbf{L} = (2 - g_J)\mathbf{J}$. In this way the Zeeman energy of Yb-ions can be written as follows

$$H_Z = g_J \sum_i \mathbf{B}\mathbf{J}_i \,, \tag{1}$$

where $\mathbf{B}$ denotes the external magnetic field multiplied by the Bohr magneton.

The Kondo exchange coupling of the rare earth ion with the conduction electrons occurs due to the hybridization of their wave functions at the ion site. The exchange integral can be written in the form (see, e.g. [18])

$$A(\mathbf{k},\mathbf{k}') = \sum_i \int \psi^*(\mathbf{r}',\mathbf{k}) \Psi_{4f}(\mathbf{r}_1,...,\mathbf{r}_i,...,\mathbf{r}_n) \frac{e^2}{|\mathbf{r}_i - \mathbf{r}'|} \psi(\mathbf{r}_i,\mathbf{k}') \Psi_{4f}(\mathbf{r}_1,...,\mathbf{r}',...,\mathbf{r}_n) d\mathbf{r}' d\mathbf{r}_1 ... d\mathbf{r}_n \,. \tag{2}$$

Here $\psi(\mathbf{r},\mathbf{k})$ is the Bloch wave function of the conduction electrons. The wave function of the 4f-electrons Ψ_{4f} is represented by the determinant constructed from the one-electron wave functions of the type $R_{4f}(r_i)Y_3^m(\mathbf{r}_i)$. Expanding the Bloch functions and $|\mathbf{r}_i - \mathbf{r}'|^{-1}$ in spherical harmonics, one can obtain $A(\mathbf{k},\mathbf{k}')$ as an expansion in multipoles. As a matter of fact the small parameter of this expansion is the value $k_F \langle r_{4f} \rangle \ll 1$, the product of the wave vector of the conduction electron at the Fermi surface and an average radius of the 4f-electron. The Kondo interaction corresponding to the zero order of this expansion is isotropic and can be written in the form [19, 20]

$$H_K = A_0 (g_J - 1) \sum_i \boldsymbol{\sigma}_i \mathbf{J}_i \,, \tag{3}$$

where $\boldsymbol{\sigma}_i$ is the spin density of the conduction electrons at the ion site. The next terms of the expansion in multipoles are $\mathbf{k}$-dependent and anisotropic (detailed calculations of them can be found in [21, 22]

and especially in [23]) but they are small and can be neglected. The same arguments can be used to derive the RKKY interaction between the Kondo ions:

$$H_{\mathrm{RKKY}} = (g_J - 1)^2 \sum_{ij} I^{ij}_{\mathrm{RKKY}} \mathbf{J}_i \mathbf{J}_j \,. \tag{4}$$

The tetragonal crystal electric field splits the ground multiplet into four Kramers doublets, each one described by the wave functions of the type $\psi_{\pm} = \sum_M C_{\pm M} \,|\pm M\rangle$ (details see in [16, 24, 25]). Within each Kramers doublet the total electronic momentum of the Yb-ion can be represented by the effective spin $S = 1/2$:

$$J^z = \gamma_{\|} S^z \,, \qquad J^{x,y} = \gamma_{\perp} S^{x,y} \,, \tag{5}$$

where $\gamma_{\|}$ and $\gamma_{\perp}$ are given by

$$\gamma_{\|} = 2\langle \psi_+ \,|\, J^z \,|\, \psi_+ \rangle \,, \qquad \gamma_{\perp} = 2\langle \psi_+ \,|\, J^x \,|\, \psi_- \rangle \,. \tag{6}$$

Since the first excited level is separated from the ground one by 17 meV (197 K) [26] and 18 meV (209 K) [27] in the cases of $YbRh_2Si_2$ and $YbIr_2Si_2$ the low temperature physics ($T \ll 200$ K) can be described by the lowest Kramers doublet. After projection onto the ground state the Zeeman energy, the Kondo interaction and the RKKY interaction take the form

$$H_{Zs} = \sum_i \Big[g_{\perp} (S_i^x B^x + S_i^y B^y) + g_{\|} S_i^z B^z \Big], \tag{7}$$

$$H_{s\sigma} = \sum_i \Big[J_{\perp} (S_i^x \sigma_i^x + S_i^y \sigma_i^y) + J_{\|} S_i^z \sigma_i^z \Big], \tag{8}$$

$$H_{\mathrm{RKKY}} = \sum_{ij} \Big[I^{ij}_{\perp} (S_i^x S_j^x + S_i^y S_j^y) + I^{ij}_{\|} S_i^z S_j^z \Big], \tag{9}$$

where $g_{\perp,\|} = g_J \gamma_{\perp,\|}$, $J_{\perp,\|} = A_0 (g_J - 1) \gamma_{\perp,\|}$, $I^{ij}_{\perp,\|} = I^{ij}_{\mathrm{RKKY}} (g_J - 1)^2 \gamma^2_{\perp,\|}$. The anisotropies of the Kondo- and RKKY interaction are evidently related to that of the *g*-factor:

$$J_{\perp} / J_{\|} = g_{\perp} / g_{\|} \,, \qquad I^{ij}_{\perp} / I^{ij}_{\|} = g^2_{\perp} / g^2_{\|} \,. \tag{10}$$

Although the experiment reveals the dispersion of the CEF levels in momentum space it does not seem to affect the principal line of our approach. Indeed, the projection onto the ground state is independent of the particular form of the wave functions $\psi_{\pm}$. It is of no importance which of the four Kramers doublet is the lowest. The only requirement to derive the Hamiltonians (7)-(9) is the large energy interval between the ground doublet and the first excited level. This condition does not appear to contradict the experimental data [17].

The kinetic energy of conduction electrons and their Zeeman energy can be written as

$$H_c = \sum_{ij\lambda} t_{ij} c^+_{i\lambda} c_{j\lambda} - \mu \sum_{i\lambda} c^+_{i\lambda} c_{i\lambda} \,, \tag{11}$$

$$H_{Z\sigma} = g_{\sigma} \sum_i \mathbf{B} \boldsymbol{\sigma}_i \,. \tag{12}$$

Here $\lambda = \pm 1$ labels the orientation of the conduction electron spin, μ is the chemical potential, g_{σ} denotes the *g*-factor of the conduction electrons. The conduction electron density is expressed in terms of the creation and annihilation operators

$$\boldsymbol{\sigma}_i = \sum_{\lambda\lambda'} \mathbf{s}_{\lambda\lambda'} c^+_{i\lambda} c_{i\lambda'} \,, \tag{13}$$

where $\mathbf{s}_{\lambda\lambda'}$ are the matrix elements of spin operators *s* = 1/2.

To study the ESR response we also added the interaction of conduction electrons and localized moments with an external alternating microwave magnetic field perpendicular to the static magnetic field. Its Hamiltonian is

$$H_{\rm mw} = g_\sigma \sum_i \mathbf{b}_{\rm mw}\boldsymbol{\sigma}_i + \sum_i \Big[g_\perp \left(S_i^x b_{\rm mw}^x + S_i^y b_{\rm mw}^y \right) + g_\parallel S_i^z b_{\rm mw}^z \Big], \tag{14}$$

where $\mathbf{b}_{\rm mw} = \mathbf{b}_0 \cos\omega t$; $\mathbf{b}_0$, ω are the amplitude and frequency of the microwave field, respectively.

Collecting all terms together we obtain the effective Hamiltonian which, after the diagonalization of the Zeeman part, takes its final form $H = H_0 + H_{s\sigma} + H_{\rm RKKY} + H_{\rm mw}$ with

$$H_0 = \sum_{ij\lambda} t_{ij} c_{i\lambda}^+ c_{j\lambda} + \sum_{i\lambda} \varepsilon^\lambda c_{i\lambda}^+ c_{i\lambda} + g_{sB} B \sum_i S_i^z \ , \tag{15}$$

$$H_{s\sigma} = \sum_{i\alpha\beta} J_{\alpha\beta} \sigma_i^\alpha S_i^\beta \ , \tag{16}$$

$$H_{\rm RKKY} = \sum_{ij\alpha\beta} I_{\alpha\beta}^{ij} S_i^\alpha S_j^\beta \ , \tag{17}$$

$$H_{\rm mw} = b_{\rm mw} \sum_i \Big[g_\sigma \sigma_i^x + g_{sb} S_i^x \Big] . \tag{18}$$

Here $\varepsilon^\lambda = -\mu + \lambda g_\sigma B/2$, $\alpha, \beta = x, y, z$, and g_{sB}, g_{sb} are the g-factors of the system of localized moments along the direction of the static and microwave magnetic fields, respectively. The explicit expressions for g_{sB}, g_{sb}, $J_{\alpha\beta}$, $I_{\alpha\beta}^{ij}$ are defined by the special orientations of $\mathbf{b}_{\rm mw}$ and $\mathbf{B}$ to the crystallographic axes. In the following it is convenient to use a complex representation of the coupling constants:

$$F_{\lambda\lambda'} = \frac{1}{2}\Big[F_{xx} - \lambda\lambda' F_{yy} + i(\lambda F_{yx} + \lambda' F_{xy}) \Big], \quad F_{\lambda z} = \frac{1}{\sqrt{2}}(F_{xz} + i\lambda F_{yz}), \quad F_{z\lambda} = \frac{1}{\sqrt{2}}(F_{zx} + i\lambda F_{zy}) \tag{19}$$

with $F = J, I$.

3. The model of spin dynamics

The ESR response due to the microwave magnetic field perturbation (18) is given by the total transverse dynamical susceptibility

$$\chi(\omega) = \frac{1}{2} \sum_{\alpha\alpha'\lambda\lambda'} \chi_{\alpha\alpha'}^{\lambda\lambda'}(\omega); \quad \alpha, \alpha' = s, \sigma; \quad \lambda, \lambda' = +, - \tag{20}$$

with partial susceptibilities $\chi_{\alpha\alpha'}^{\lambda\lambda'}(\omega)$:

$$\begin{aligned} \chi_{\rm ss}^{\lambda\lambda'} &= -g_{sb}^2 \left\langle\!\left\langle S^\lambda \middle| S^{\lambda'} \right\rangle\!\right\rangle, \qquad \chi_{s\sigma}^{\lambda\lambda'} = -g_{sb} g_\sigma \left\langle\!\left\langle S^\lambda \middle| \sigma^{\lambda'} \right\rangle\!\right\rangle, \\ \chi_{\sigma s}^{\lambda\lambda'} &= -g_{sb} g_\sigma \left\langle\!\left\langle \sigma^{\lambda'} \middle| S^\lambda \right\rangle\!\right\rangle, \quad \chi_{\sigma\sigma}^{\lambda\lambda'} = -g_\sigma^2 \left\langle\!\left\langle \sigma^\lambda \middle| \sigma^{\lambda'} \right\rangle\!\right\rangle. \end{aligned} \tag{21}$$

Here $\left\langle\!\left\langle A \middle| B \right\rangle\!\right\rangle$ is the Fourier transform of a retarded Green function

$$\left\langle\!\left\langle A \middle| B \right\rangle\!\right\rangle = -i \int_0^\infty dt \exp(i\omega t) \left\langle \big[A(t), B \big] \right\rangle, \tag{22}$$

S, **σ** are the total spin operators of Yb-ions and conduction electrons, $S^\lambda = (S^x + i\lambda S^y)/\sqrt{2}$, $\sigma^\lambda = (\sigma^x + i\lambda\sigma^y)/\sqrt{2}$, respectively, and $\langle \ldots \rangle$ means the statistical average at temperature T.

The definitions (21) and (22) imply the symmetry relations

$$\left(\chi^{\lambda\lambda'}_{\alpha\alpha'}(\omega)\right)^* = \chi^{\bar{\lambda}'\lambda}_{\alpha'\alpha}(\omega^*), \qquad \chi^{\lambda\lambda'}_{\alpha\alpha'}(\omega) = \chi^{\lambda'\lambda}_{\alpha'\alpha}(-\omega), \tag{23}$$

where $\bar{\lambda} = -\lambda$; ω is extended to the complex plane.

It is usually accepted to take into account only resonant terms of the type χ^{-+} dropping χ^{+-} and χ^{++} as nonresonant ones, but this is not generally correct. Thus, if one of interacting subsystems (for example, conduction electrons) is in the thermodynamical equilibrium, the spin dynamics is described by the partial susceptibilities $\chi^{\lambda\lambda'}_{ss}$ of the well known form

$$\chi^{-+}_{ss} \sim (\omega - \omega_s + i\Gamma_{ss})^{-1}, \qquad \chi^{+-}_{ss} \sim (\omega + \omega_s - i\Gamma_{ss})^{-1}, \tag{24}$$

where ω_s is the resonant frequency of Yb-ions and Γ_{ss} is the Korringa relaxation rate. χ^{-+}_{ss} and χ^{+-}_{ss} can be considered as resonant and nonresonant parts of the total susceptibility, respectively. For an isotropic system χ^{++}_{ss} and χ^{--}_{ss} vanish and the ESR response is really given by the resonant part χ^{-+}_{ss}. In the case of an anisotropic interaction χ^{++}_{ss} and χ^{--}_{ss} may happen to be non-zero giving additional contributions to the total susceptibility. Although χ^{++}_{ss} and χ^{--}_{ss} are small as compared with χ^{-+}_{ss} they are of the same order of magnitude as the relaxation rate Γ_{ss} (the second order in the Kondo interaction), which can result in some corrections to the ESR linewidth.

The problem of nonresonant terms becomes even more obscure when the subsystems of conduction electrons and localized moments are strongly coupled. Then the total susceptibility is determined by the collective spin motion and the partial susceptibilities are found as solutions of coupled equations which may lead to the form quite different from (24). Nevertheless, we can still consider the partial susceptibilities $\chi^{-+}_{\alpha\alpha'}$ as resonant if $\chi^{++}_{\alpha\alpha'}, \chi^{--}_{\alpha\alpha'} = 0$ and the equations of motion for $\chi^{-+}_{\alpha\alpha'}$ and $\chi^{+-}_{\alpha\alpha'}$ are not coupled with each other. However, for an arbitrary anisotropy of the Kondo interaction with non-vanishing $\chi^{++}_{\alpha\alpha'}$, when all partial susceptibilities are coupled in a whole system of equations, it is rather difficult to separate resonant and nonresonant contributions in advance. In any case, it is worthwhile to confirm a tentative approximation with more accurate analysis.

The calculation of partial susceptibilities leads to a set of coupled equations, which are convenient to write in matrix form

$$\begin{pmatrix} a^{-+}_{ss} & a^{-+}_{s\sigma} & a^{--}_{ss} & a^{--}_{s\sigma} \\ a^{-+}_{\sigma s} & a^{-+}_{\sigma\sigma} & a^{--}_{\sigma s} & a^{--}_{\sigma\sigma} \\ a^{++}_{ss} & a^{++}_{s\sigma} & a^{+-}_{ss} & a^{+-}_{s\sigma} \\ a^{++}_{\sigma s} & a^{++}_{\sigma\sigma} & a^{+-}_{\sigma s} & a^{+-}_{\sigma\sigma} \end{pmatrix} \times \begin{pmatrix} \chi^{-+}_{ss} & \chi^{-+}_{s\sigma} & \chi^{--}_{ss} & \chi^{--}_{s\sigma} \\ \chi^{-+}_{\sigma s} & \chi^{-+}_{\sigma\sigma} & \chi^{--}_{\sigma s} & \chi^{--}_{\sigma\sigma} \\ \chi^{++}_{ss} & \chi^{++}_{s\sigma} & \chi^{+-}_{ss} & \chi^{+-}_{s\sigma} \\ \chi^{++}_{\sigma s} & \chi^{++}_{\sigma\sigma} & \chi^{+-}_{\sigma s} & \chi^{+-}_{\sigma\sigma} \end{pmatrix} = \begin{pmatrix} P^{-+}_{ss} & 0 & 0 & 0 \\ 0 & P^{-+}_{\sigma\sigma} & 0 & 0 \\ 0 & 0 & P^{+-}_{ss} & 0 \\ 0 & 0 & 0 & P^{+-}_{\sigma\sigma} \end{pmatrix} \tag{25}$$

with

$$\begin{aligned} &a^{\bar{\lambda}\lambda}_{\alpha\alpha} = \omega - \lambda\omega_\alpha + \Sigma^{\bar{\lambda}\lambda}_{\alpha\alpha}, && a^{\lambda\lambda}_{\alpha\alpha} = \Sigma^{\lambda\lambda}_{\alpha\alpha}, \\ &a^{\lambda\lambda'}_{s\sigma} = \frac{g_{sb}}{g_\sigma}\left(\lambda\langle S^z\rangle J_{\lambda'\lambda} - \Sigma^{\lambda\lambda'}_{s\sigma}\right), && a^{\lambda\lambda'}_{\sigma s} = \frac{g_\sigma}{g_{sb}}\left(\lambda\langle\sigma^z\rangle J_{\lambda\lambda'} - \Sigma^{\lambda\lambda'}_{\sigma s}\right), \\ &P^{\bar{\lambda}\lambda}_{ss} = \lambda g^2_{sb}\langle S^z\rangle, && P^{\bar{\lambda}\lambda}_{\sigma\sigma} = \lambda g^2_{sb}\langle\sigma^z\rangle, \qquad \alpha = s,\sigma; \quad \lambda = +,-. \end{aligned} \tag{26}$$

Here $J_{\lambda\lambda'}$ is defined by (19),

$$\langle S^z\rangle = -g_{sB}B/4(T + T_W) + O(J), \qquad \langle\sigma^z\rangle = -g_\sigma\rho B/2 + O(J), \tag{27}$$

ω_s and ω_σ are the resonant frequencies of Yb-ions and conduction electrons, respectively, containing

first order Knight shifts due to the Kondo- and RKKY interaction:

$$\omega_s = g_{sB}B + J_{zz}\langle\sigma^z\rangle + 4T_W\langle S^z\rangle, \qquad \omega_\sigma = g_\sigma B + J_{zz}\langle S^z\rangle. \tag{28}$$

T_W denotes the Weiss temperature which originates from the RKKY interaction in a molecular field approximation:

$$T_W = \frac{1}{4}\sum_i I_{zz}^{ij}, \tag{29}$$

and ρ denotes the conduction electron density at the Fermi surface.

The symmetry relations for the partial susceptibilities (23) and the equations (25), (26) give the symmetry relations for the kinetic coefficients $\Sigma_{\alpha\alpha'}^{\lambda\lambda'}$:

$$\begin{aligned} \left(\Sigma_{\alpha\alpha}^{\lambda\lambda'}(\omega)\right)^* &= -\lambda\lambda'\Sigma_{\alpha\alpha}^{\bar{\lambda}'\lambda}(\omega^*), \qquad \left(\Sigma_{s\sigma}^{\lambda\lambda'}(\omega)\right)^* = -\lambda\lambda'\frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Sigma_{\sigma s}^{\bar{\lambda}'\lambda}(\omega^*), \\ \Sigma_{\alpha\alpha}^{\lambda\lambda'}(-\omega) &= \lambda\lambda'\Sigma_{\alpha\alpha}^{\lambda'\lambda}(\omega), \qquad \Sigma_{s\sigma}^{\lambda\lambda'}(-\omega) = \lambda\lambda'\frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Sigma_{\sigma s}^{\lambda'\lambda}(\omega). \end{aligned} \tag{30}$$

The partial relaxation rates are usually written as the imaginary parts of corresponding kinetic coefficients $\Gamma_{\alpha\alpha'}^{\lambda\lambda'} = \mathrm{Im}\left[\Sigma_{\alpha\alpha'}^{\lambda\lambda'}(\omega + i0)\right]$. However, if the Kondo- and RKKY exchange constants $J_{\lambda\lambda'}$ and $I_{\lambda\lambda'}$ are complex it is more convenient to use the following definition:

$$\Gamma_{\alpha\alpha'}^{\lambda\lambda'} = \frac{1}{2}\left|\Sigma_{\alpha\alpha'}^{\lambda\lambda'}(\omega + i0) - \Sigma_{\alpha\alpha'}^{\lambda\lambda'}(\omega - i0)\right|. \tag{31}$$

When the Kondo interaction is isotropic or the static magnetic field is oriented parallel to the crystal symmetry axis the matrix elements $a_{\alpha\alpha'}^{\lambda\lambda}$ and $\chi_{\alpha\alpha'}^{\lambda\lambda}$ vanish and the equation (25) is divided into two independent 2-by-2 matrix equations

$$\begin{pmatrix} a_{ss}^{\bar{\lambda}\lambda} & a_{s\sigma}^{\bar{\lambda}\lambda} \\ a_{\sigma s}^{\bar{\lambda}\lambda} & a_{\sigma\sigma}^{\bar{\lambda}\lambda} \end{pmatrix} \times \begin{pmatrix} \chi_{ss}^{\bar{\lambda}\lambda} & \chi_{s\sigma}^{\bar{\lambda}\lambda} \\ \chi_{\sigma s}^{\bar{\lambda}\lambda} & \chi_{\sigma\sigma}^{\bar{\lambda}\lambda} \end{pmatrix} = \begin{pmatrix} P_{ss}^{\bar{\lambda}\lambda} & 0 \\ 0 & P_{\sigma\sigma}^{\bar{\lambda}\lambda} \end{pmatrix} \tag{32}$$

with $\lambda = +, -$ corresponding to resonant and nonresonant partial susceptibilities, respectively. In these cases the nonresonant contributions can well be ignored. Equations of the type (32) are usually used to study the collective spin motion of localized moments and conduction electrons [28, 29]. For an arbitrary orientation of the static magnetic field to the crystal symmetry axis $a_{\alpha\alpha'}^{\lambda\lambda}$ and $\chi_{\alpha\alpha'}^{\lambda\lambda}$ may happen to be non-zero, what does not allow us to reduce the matrix equation (25) to two independent equations of the form (32). In this case resonant and nonresonant partial susceptibilities $\chi_{\alpha\alpha'}^{-+}$ and $\chi_{\alpha\alpha'}^{+-}$ are coupled in the collective spin motion and the actual resonant contribution to the total susceptibility is found as a solution of the whole system (25).

The kinetic coefficients $\Sigma_{ss}^{\bar{\lambda}\lambda}$ and $\Sigma_{\sigma\sigma}^{\bar{\lambda}\lambda}$ describe the well known Korringa and Overhauser relaxations (Yb-ions relax to the conduction electrons being in the thermodynamical equilibrium and vice versa). Two additional coefficients $\Sigma_{s\sigma}^{\bar{\lambda}\lambda}$ and $\Sigma_{\sigma s}^{\bar{\lambda}\lambda}$ couple the equations of motion for the transverse magnetization of localized moments and conduction electrons. The new kinetic coefficients $\Sigma_{\alpha\alpha'}^{\lambda\lambda}$ provide the coupling between the resonant and nonresonant parts of the total magnetization. Besides, for a correct analysis of a stationary solution one has to take into account the spin relaxation of Kondo ions and conduction electrons to the thermal bath ("lattice"). Correspondingly, the kinetic coefficients $\Sigma_{ss}^{\bar{\lambda}\lambda}$ and $\Sigma_{\sigma\sigma}^{\bar{\lambda}\lambda}$ should be replaced with $\Sigma_{ss}^{\bar{\lambda}\lambda} + \Sigma_{sL}$ and $\Sigma_{\sigma\sigma}^{\bar{\lambda}\lambda} + \Sigma_{\sigma L}$, respectively. The poles of the total susceptibility are determined by the condition

$$\det\left\|a_{\alpha\alpha'}^{\lambda\lambda'}\right\| = 0 \tag{33}$$

leading to four complex roots, only two of which are resonant. Their real parts represent resonant frequencies and their imaginary parts represent the corresponding relaxation rates. Among the two resonant solutions we are interested in a pole close to the Kondo-ion resonant frequency, which describes the collective spin motion with the narrow ESR linewidth. Another resonant solution (close to the conduction electron resonant frequency ω_σ) gives the imaginary part too large to be observable by the ESR. The partial susceptibilities are found as the solutions of the system (25): $\chi_{\alpha\alpha'}^{\lambda\lambda'} = (a^{-1})_{\alpha\alpha'}^{\lambda\lambda'} P_{\alpha'\alpha'}^{\bar{\lambda}'\lambda'}$. One can see, that in the case of non-zero coupling terms $\chi_{\alpha\alpha'}^{\lambda\lambda}$ all parts of the total susceptibility, including the terms considered earlier as nonresonant, have the common resonant pole given by the equation (33).

The coupling between conduction electrons and localized moments is especially important if the relaxation rate of conduction electrons toward the Kondo ions is much faster than to the lattice and the resonant frequencies are close to one another ("bottleneck" regime)

$$\Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} \gg \Gamma_{\sigma L},\ \left|\omega_s - \omega_\sigma\right|. \tag{34}$$

In this research the spin kinetics is studied for two special configurations of the static and microwave magnetic fields at the temperatures high compared with the static field ($T > B$). At first we find the kinetic coefficients up to the second order in the Kondo interaction by means of the functional derivative method [30, 9], then the perturbational approach is improved with the Anderson's "poor man's scaling" technique [31].

4. The ESR parameters to the second order in the Kondo interaction

In the case of the static magnetic field oriented parallel to the crystal symmetry axis *c* with the microwave field lying in the crystallographic plane (fig. 1a) the parameters of the effective Hamiltonian (15)-(18) takes the form

$$g_{sB} = g_\parallel, \quad g_{sb} = g_\perp, \tag{35}$$

$$J_{zz} = J_\parallel, \quad J_{\lambda\bar{\lambda}} = J_\perp, \quad J_{\lambda\lambda} = J_{\lambda z} = J_{z\lambda} = 0, \tag{36}$$

$$I_{zz} = I_\parallel, \quad I_{\lambda\bar{\lambda}} = I_\perp, \quad I_{\lambda\lambda} = I_{\lambda z} = I_{z\lambda} = 0. \tag{37}$$

The calculation of the kinetic coefficients up to the second order in the Kondo interaction leads to the vanishing coupling between the resonant and nonresonant terms of the total susceptibility: $a_{\alpha\alpha'}^{\lambda\lambda}, \chi_{\alpha\alpha'}^{\lambda\lambda} = 0$. As it was mentioned above, in this case the resonant and nonresonant partial susceptibilities $\chi_{\alpha\alpha'}^{-+}$ and $\chi_{\alpha\alpha'}^{+-}$ can be considered separately and the resonant pales of the total susceptibility are determined by the terms of the type $\chi_{\alpha\alpha'}^{-+}$. The partial relaxation rates are given by

$$\begin{aligned}
&\Gamma_{ss}^{\bar{\lambda}\lambda} = \frac{\pi}{2}\frac{\omega}{\omega_s} T\rho^2 (J_\perp^2 + J_\parallel^2), \qquad \Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z\rangle}{\langle \sigma^z\rangle}\Gamma_{ss}^{\bar{\lambda}\lambda},\\
&\Gamma_{\sigma s}^{\bar{\lambda}\lambda} = \pi\frac{\omega}{\omega_s} T\rho^2 J_\perp J_\parallel, \qquad \Gamma_{s\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z\rangle}{\langle \sigma^z\rangle}\Gamma_{\sigma s}^{\bar{\lambda}\lambda},\\
&\Gamma_{\alpha\alpha'}^{\lambda\lambda} = 0, \qquad \frac{\langle S^z\rangle}{\langle \sigma^z\rangle} = \frac{g_\parallel}{2g_\sigma\rho(T + T_W)}.
\end{aligned} \tag{38}$$

The Overhauser relaxation rate $\Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda}$ is seen to be much greater than the Korringa relaxation rate $\Gamma_{ss}^{\bar{\lambda}\lambda}$ due to the large value of the ratio $\langle S^z\rangle / \langle \sigma^z\rangle \sim 1/\rho T$.

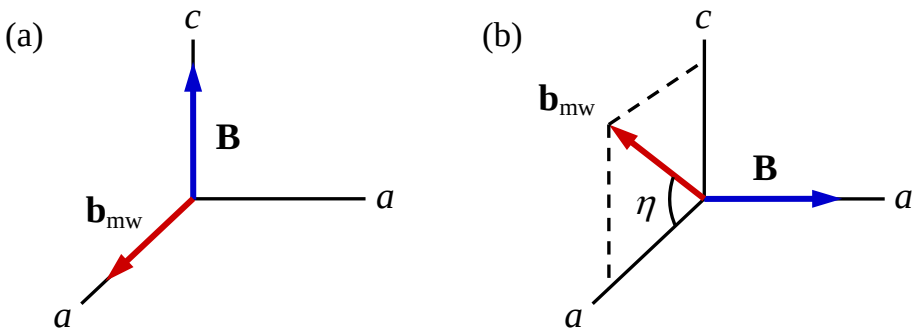

Figure 1. Two special configurations of magnetic fields considered in this paper. $\mathbf{B}$ and $\mathbf{b}_{\rm mw}$ are the static and microwave magnetic fields, respectively.

The resonant poles of the total susceptibility are found from the condition $\det\left\|a^{-+}_{\alpha\alpha'}\right\|=0$ (the full determinant in equation (33) is factorized as $\det\left\|a^{\lambda\lambda'}_{\alpha\alpha'}\right\|=\det\left\|a^{-+}_{\alpha\alpha'}\right\|\det\left\|a^{+-}_{\alpha\alpha'}\right\|$). Under the condition of the strong bottleneck regime (34) the imaginary part of the resonant pole corresponding to the relaxation rate of the collective spin mode reads as follows:

$$\Gamma^{\|}_{\rm coll}=\Gamma^{\|}_{sL}+\tilde{\Gamma}^{\|}_{\sigma L}+\tilde{\Gamma}^{\|}_{ss},\qquad \tilde{\Gamma}^{\|}_{\sigma L}=\Gamma_{\sigma L}\frac{\Gamma^{-+}_{\sigma s}\Gamma^{-+}_{s\sigma}}{(\Gamma^{-+}_{\sigma\sigma})^2},\qquad \tilde{\Gamma}^{\|}_{ss}=\Gamma^{-+}_{ss}-\frac{\Gamma^{-+}_{\sigma s}\Gamma^{-+}_{s\sigma}}{\Gamma^{-+}_{\sigma\sigma}}. \tag{39}$$

The substitution of (38) into (39) yields the following expressions for an effective Korringa relaxation rate $\tilde{\Gamma}^{\|}_{ss}$ and an effective relaxation rate of conduction electrons to the lattice $\tilde{\Gamma}^{\|}_{\sigma L}$:

$$\tilde{\Gamma}^{\|}_{ss}=\frac{\pi}{2}T\rho^2\frac{\left(J^2_{\perp}-J^2_{\|}\right)^2}{J^2_{\perp}+J^2_{\|}},\qquad \tilde{\Gamma}^{\|}_{\sigma L}=\Gamma_{\sigma L}\frac{\langle\sigma^z\rangle}{\langle S^z\rangle}\left(\frac{2J_{\perp}J_{\|}}{J^2_{\perp}+J^2_{\|}}\right)^2. \tag{40}$$

For an isotropic system we have the well-known result [28]: the ESR linewidth in the bottleneck regime is greatly narrowed due to the conservation of the total magnetic moment (its operator commutes with the isotropic Kondo interaction and the latter disappears from the effective relaxation rate). In the opposite case of a strongly anisotropic Kondo interaction ($J_{\perp}\gg J_{\|}$) our expressions do not show any sufficient narrowing of the ESR linewidth in the bottleneck regime, which coincides mainly with the results of the work [29]. Concerning the effective relaxation rate $\tilde{\Gamma}^{\|}_{\sigma L}$ it is greatly reduced against $\Gamma_{\sigma L}$ by the small value of the ratio $\langle\sigma^z\rangle/\langle S^z\rangle\sim\rho T$ and shows the linear temperature dependence similar to the Korringa relaxation.

Next we consider the perpendicular orientation of the static magnetic field toward the crystal symmetry axis c with the microwave field directed at an arbitrary angle η to the crystallographic plane (fig. 1b). The parameters of the effective Hamiltonian are written as follows

$$g_{sB}=g_{\perp},\qquad g_{sb}=\sqrt{g^2_{\perp}\cos^2\eta+g^2_{\|}\sin^2\eta}\,, \tag{41}$$

$$J_{zz}=J_{\perp},\qquad J_{\lambda\lambda'}=\frac{1}{2}(J_{\perp}-\lambda\lambda' J_{\|})\exp\{i(\lambda\eta+\lambda'\psi)\},\qquad J_{\lambda z}=J_{z\lambda}=0, \tag{42}$$

$$I_{zz}=I_{\perp},\qquad I_{\lambda\lambda'}=\frac{1}{2}(I_{\perp}-\lambda\lambda' I_{\|})\exp\{i(\lambda+\lambda')\psi\},\qquad I_{\lambda z}=I_{z\lambda}=0. \tag{43}$$

Here ψ is the angle between a new quantization axis x and the crystallographic plane:

$$\tan\psi = \frac{g_\parallel}{g_\perp}\tan\eta\,. \tag{44}$$

The partial relaxation rates corresponding to the individual spin motions of the resonant and nonresonant components of the total magnetization are of the form

$$\begin{aligned}
&\Gamma_{ss}^{\bar\lambda\lambda} = \frac{\pi}{4}\frac{\omega}{\omega_s}T\rho^2(3J_\perp^2 + J_\parallel^2), \qquad \Gamma_{\sigma\sigma}^{\bar\lambda\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{ss}^{\bar\lambda\lambda},\\
&\Gamma_{\sigma s}^{\bar\lambda\lambda} = \frac{\pi}{2}\frac{\omega}{\omega_s}T\rho^2 J_\perp(J_\perp + J_\parallel), \qquad \Gamma_{s\sigma}^{\bar\lambda\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{\sigma s}^{\bar\lambda\lambda},\\
&\frac{\langle S^z\rangle}{\langle\sigma^z\rangle} = \frac{g_\perp}{2g_\sigma\rho(T+T_{\rm W})}.
\end{aligned} \tag{45}$$

Besides, for this particular configuration of the static and microwave magnetic fields we obtain the additional kinetic coefficients providing the coupling between the resonant and nonresonant components:

$$\begin{aligned}
&\Gamma_{ss}^{\lambda\lambda} = \frac{\pi}{4}\frac{\omega}{\omega_s}T\rho^2(J_\perp^2 - J_\parallel^2), \qquad \Gamma_{\sigma\sigma}^{\lambda\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{ss}^{\lambda\lambda},\\
&\Gamma_{\sigma s}^{\lambda\lambda} = \frac{\pi}{2}\frac{\omega}{\omega_s}T\rho^2 J_\perp(J_\perp - J_\parallel), \qquad \Gamma_{s\sigma}^{\lambda\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{\sigma s}^{\lambda\lambda}.
\end{aligned} \tag{46}$$

In the case of the strongly anisotropic Kondo interaction ($J_\perp \gg J_\parallel$) the coupling between the resonant and nonresonant components become comparable with the coupling between the subsystems of the conduction and localized moments: the kinetic coefficients (45) and (46) are of the same order of magnitude. In this case the resonant and nonresonant terms can not be considered separately and the resonant poles of the total susceptibility (20) are determined by the equation (33).

Under the condition of the bottleneck regime the relaxation rate of the collective spin mode can be written as geometrical mean of two relaxation rates:

$$\Gamma_{\rm coll} = \sqrt{\Gamma^{(1)}\Gamma^{(2)}}, \quad \Gamma^{(i)} = \Gamma_{sL} + \tilde\Gamma_{\sigma L}^{(i)} + \tilde\Gamma_{ss}^{(i)}, \quad \tilde\Gamma_{\sigma L}^{(i)} = \Gamma_{\sigma L}\frac{\Gamma_{\sigma s}^{(i)}\Gamma_{s\sigma}^{(i)}}{(\Gamma_{\sigma\sigma}^{(i)})^2}, \quad \tilde\Gamma_{ss}^{(i)} = \Gamma_{ss}^{(i)} - \frac{\Gamma_{\sigma s}^{(i)}\Gamma_{s\sigma}^{(i)}}{\Gamma_{\sigma\sigma}^{(i)}}, \quad i = 1,2. \tag{47}$$

Here $\Gamma_{\alpha\beta}^{(i)}$ represents the partial relaxation rate renormalized by the coupling between the resonant and nonresonant components of the total magnetization:

$$\Gamma_{\alpha\alpha'}^{(1)} = \Gamma_{\alpha\alpha'}^{-+} - \Gamma_{\alpha\alpha'}^{--}, \quad \Gamma_{\alpha\alpha'}^{(2)} = \Gamma_{\alpha\alpha'}^{-+} + \Gamma_{\alpha\alpha'}^{--}, \quad \alpha,\alpha' = s,\sigma. \tag{48}$$

The expressions (47) and (48) are formally applicable to either of two configurations of the static and microwave magnetic fields considered in this research. In the case of the static field oriented parallel to the *c*-axis the coupling between the resonant and nonresonant components vanishes and $\Gamma^{(1)} = \Gamma^{(2)} = \Gamma_{\rm coll}^{\parallel}$. When the static magnetic field is perpendicular to the crystal symmetry axis the partial relaxation rates $\Gamma_{\alpha\beta}^{(1)}$, after substitution of (45) and (46) into (48), take the form

$$\Gamma_{ss}^{(1)} = \Gamma_{\sigma s}^{(1)} = \pi\frac{\omega}{\omega_s}T\rho^2 J_\perp^2, \qquad \Gamma_{\sigma\sigma}^{(1)} = \Gamma_{s\sigma}^{(1)} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{ss}^{(1)}. \tag{49}$$

The effective Korringa relaxation rate $\tilde\Gamma_{ss}^{(1)}$, the effective relaxation rate of conduction electrons to the lattice $\tilde\Gamma_{\sigma L}^{(1)}$ and the relaxation rate $\Gamma^{(1)}$ are given by

$$\tilde{\Gamma}_{ss}^{(1)} = 0, \quad \tilde{\Gamma}_{\sigma L}^{(1)} = \Gamma_{\sigma L} \frac{\langle \sigma^z \rangle}{\langle S^z \rangle}, \quad \Gamma^{(1)} = \Gamma_{sL}^{\perp} + \tilde{\Gamma}_{\sigma L}^{(1)}. \tag{50}$$

The quantity $\Gamma^{(1)}$ can obviously be identified as the relaxation rate of the collective spin mode in the case of an isotropic system, when the Kondo interaction makes no contribution to the effective relaxation rate.

Similar operations with $\Gamma_{\alpha\alpha'}^{(2)}$, $\tilde{\Gamma}_{ss}^{(2)}$, $\tilde{\Gamma}_{\sigma L}^{(2)}$ leads to almost the same results as those obtained for the case of the parallel orientation of the static magnetic field to the *c*-axis (38)-(40):

$$\tilde{\Gamma}_{ss}^{(2)} = \tilde{\Gamma}_{ss}^{\parallel}, \quad \tilde{\Gamma}_{\sigma L}^{(2)} = \tilde{\Gamma}_{\sigma L}^{\parallel} \frac{g_{\parallel}}{g_{\perp}}, \quad \Gamma^{(2)} = \Gamma_{sL}^{\perp} + \tilde{\Gamma}_{\sigma L}^{(2)} + \tilde{\Gamma}_{ss}^{(2)}. \tag{51}$$

In this way, when the static magnetic field is perpendicular to the crystal symmetry axis, the relaxation rate of the collective spin mode is represented as

$$\Gamma_{\text{coll}}^{\perp} = \sqrt{\Gamma^{(1)}\Gamma^{(2)}} \approx \sqrt{\Gamma_{\text{coll}}^{\text{isotr}}\, \Gamma_{\text{coll}}^{\parallel}} \tag{52}$$

with $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by (50) and (51). The ESR linewidth is partly narrowed because the large contribution of the Kondo interaction to $\Gamma^{(2)} \approx \Gamma_{\text{coll}}^{\parallel}$ is reduced by the small values of the spin lattice relaxation rates Γ_{sL} and $\tilde{\Gamma}_{\sigma L}$ (if we put $\Gamma_{sL} = \Gamma_{\sigma L} = 0$ the relaxation rate $\Gamma_{\text{coll}}^{\perp}$ would be equal to zero too). This result can be predicted from general considerations: one of the transverse components of total spin operator ($S^y + \sigma^y$) commutes with the effective Kondo interaction (16) when the static magnetic field is perpendicular to the *c*-axis.

It is interesting to follow the variation of the ESR parameter with the orientation of the magnetic field (angle η in fig. 1b). The ESR linewidth which is associated with the relaxation rate $\Gamma_{\text{coll}}^{\perp}$ (52) and the resonant *g*-factor close to $g_{sB} = g_{\perp}$ are evidently independent of the angle η. This result agrees with experimental data on $YbIr_2Si_2$ [14].

Concerning the ESR intensity, the situation is quite different. The total absorption intensity is determined by integrating the absorbed power of the microwave magnetic field $\mathbf{b}_{\text{mw}}$ which, in its turn, is related to the transverse dynamical susceptibility. The partial susceptibilities $\chi_{\sigma\sigma}^{\lambda\lambda'}$, $\chi_{\sigma s}^{\lambda\lambda'}$, $\chi_{s\sigma}^{\lambda\lambda'}$ are negligible as compared with $\chi_{ss}^{\lambda\lambda'}$ due to the small value of the ratio $\langle \sigma^z \rangle / \langle S^z \rangle$, hence, the main contribution to the ESR intensity is made by the term $\chi_{ss}^{\lambda\lambda'}$. Since the kinetic coefficients $\Sigma_{\alpha\alpha'}^{\lambda\lambda'}$ and the resonant frequency do not depend on the orientation of the microwave magnetic field, the only parameter of the dynamical susceptibility depending on the angle η is the *g*-factor g_{sb}. The angular dependence of the ESR intensity $I(\eta)$ can be derived from the relation $I(\eta)/I(0) = g_{sb}^2(\eta)/g_{sb}^2(0)$ without integrating the absorbed power:

$$I(\eta) = I(0)\left[1 - \left(1 - \frac{g_{\parallel}^2}{g_{\perp}^2}\right)\sin^2\eta\right]. \tag{53}$$

The equation (53) reveals a strong angular dependence of the ESR intensity: $g_{\perp}^2/g_{\parallel}^2 \approx 400$ and $g_{\perp}^2/g_{\parallel}^2 \approx 14$ for the cases of $YbRh_2Si_2$ and $YbIr_2Si_2$, respectively. These values do not agree with the ratio $I(0)/I(\pi/2) \approx 2$ experimentally observed for both compounds. It seems to point out that our consideration does not take into account some factors that can reduce the large value of the ratio $g_{\perp}/g_{\parallel}$.

5. The renormalization of the ESR parameters

At low temperatures the second order of the standart perturbation expansion is not sufficient, especially in the case of an antiferromagnetic coupling ($J > 0$): the higher order calculations show the logarithmic divergencies of the type $\ln(T/W)$, where W is a conduction electron band width [9, 32]. The perturbation technique can be improved on the basis of the "poor man's scaling" method proposed by Anderson [31]. The main idea of this approach is to take into account the effect of the high energy excitations on the low energy physics by a renormalization of coupling constants. The original Kondo interaction $H_{s\sigma}$ (16) is projected on to the low energy states yielding a Hamiltonian $\tilde{H}_{s\sigma}$ with new Kondo couplings $\tilde{J}_{\perp}$ and $\tilde{J}_{\parallel}$ (details see in [9, 10]). The renormalized parameters $U_{\perp} = \widetilde{\rho J_{\perp}}$ and $U_{\parallel} = \widetilde{\rho J_{\parallel}}$ become temperature dependent:

$$U_{\perp} = \bar{U}/\sin\varphi, \qquad U_{\parallel} = \bar{U}\cot\varphi. \tag{54}$$

Here $\bar{U} = \rho\sqrt{J_{\perp}^2 - J_{\parallel}^2}$, $\varphi = \bar{U}\ln(T/T_{\mathrm{GK}})$, the abbreviation “GK” indicates the Kramers ground state and T_{GK} denotes a characteristic temperature given as follows

$$T_{\mathrm{GK}} = W\exp\left[-\frac{1}{\bar{U}}\arccos\left(\frac{g_{\parallel}}{g_{\perp}}\right)\right]. \tag{55}$$

The quantities T_{GK} and $\bar{U}$ are scaling invariant which do not change with renormalizing the Hamiltonian $H_{s\sigma}$. Although the actual structure of the Fermi surface is rather complicated [33] we use the simplest approximation of the constant density of states for conduction electrons keeping in mind to study that temperature-magnetic field region where the heavy fermions are absent and the peculiarities of the Fermi surface are not important.

We also suppose the anisotropy of the Zeeman energy to be the same as that of the Kondo interaction independently of the scaling procedure. The relation $g_{\perp} / g_{\parallel} = J_{\perp} / J_{\parallel}$ (see (10)) converts, after renormalization, to

$$\tilde{g}_{\perp} / \tilde{g}_{\parallel} = U_{\perp} / U_{\parallel}. \tag{56}$$

Using the "poor man's scaling" method one can find the kinetic coefficients renormalized by the high energy excitations. In the case of the parallel orientation of the static magnetic field to the crystal symmetry axis the partial relaxation rates take the form

$$\begin{aligned}
&\Gamma_{ss}^{\bar{\lambda}\lambda} = \pi\frac{\omega}{\omega_s}\bar{U}^2 T\left(\cot^2\varphi + \frac{1}{2}\right), && \Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{\mathrm{ss}}^{\bar{\lambda}\lambda},\\
&\Gamma_{\sigma s}^{\bar{\lambda}\lambda} = \pi\frac{\omega}{\omega_s}\bar{U}^2 T\frac{\cos\varphi}{\sin^2\varphi}, && \Gamma_{s\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z\rangle}{\langle\sigma^z\rangle}\Gamma_{\sigma s}^{\bar{\lambda}\lambda},\\
&\Gamma_{\alpha\alpha'}^{\lambda\lambda} = 0, && \frac{\langle S^z\rangle}{\langle\sigma^z\rangle} = \frac{g_{\parallel}}{2g_{\sigma}\rho(T+T_{\mathrm{W}})}.
\end{aligned} \tag{57}$$

Substituting the renormalized relaxation rates (57) into (39) we obtain the relaxation rate of the collective spin mode

$$\Gamma_{\mathrm{coll}}^{\parallel} = \Gamma_{sL}^{\parallel} + \tilde{\Gamma}_{\sigma L}^{\parallel} + \tilde{\Gamma}_{ss}^{\parallel}, \qquad \tilde{\Gamma}_{ss}^{\parallel} = \frac{\pi}{2}T\bar{U}^2\frac{\sin^2\varphi}{1+\cos^2\varphi}, \qquad \tilde{\Gamma}_{\sigma L}^{\parallel} = \Gamma_{\sigma L}\frac{\langle\sigma^z\rangle}{\langle S^z\rangle}\left(\frac{2\cos\varphi}{1+\cos^2\varphi}\right)^2. \tag{58}$$

It is interesting to analyze the asymptotic behavior of the relaxation rates (57) and (58) upon lowering the temperature to T_{GK} ($\varphi \ll 1$). The partial relaxation rates logarithmically diverge at $T \to T_{\mathrm{GK}}$: to the

leading order in logarithmic terms they are of the form

$$\Gamma_{ss}^{\bar{\lambda}\lambda} = \Gamma_{\sigma s}^{\bar{\lambda}\lambda} = \pi \frac{\omega}{\omega_s} \frac{T}{\ln^2(T/T_{\rm GK})}, \qquad \Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} = \Gamma_{s\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{ss}^{\bar{\lambda}\lambda}. \tag{59}$$

At first glance, these results confirm the common belief that the ESR linewidth of Kondo ions (as well as conduction electrons) is expected to be too large for its detection. However, the coupling between the two systems makes the situation quite different. The effective Korringa relaxation rate is greatly reduced as compared with the second order result (40) (apart from the divergent partial relaxation rates (59)) and goes to zero at $T \to T_{\rm GK}$:

$$\tilde{\Gamma}_{ss}^{\parallel} = \frac{\pi}{4} T \bar{U}^4 \ln^2(T/T_{\rm GK}). \tag{60}$$

Although the Kondo interaction is strongly anisotropic, the divergent parts of different kinetic coefficients cancel each other in the collective spin mode due to the existence of the common energy scale $T_{\rm GK}$ regulating their temperature dependence at $T \to T_{\rm GK}$. From the point of view of a renormalization formalism the Kondo interaction tends to be isotropic as affected by the scaling procedure. The renormalized effective relaxation rate of conduction electrons to the lattice $\tilde{\Gamma}_{\sigma L}^{\parallel}$ differs from the second order result (40) but slightly; its reduction is due to the small value of the ratio $\langle \sigma^z \rangle / \langle S^z \rangle \sim \rho T$ rather than to the Kondo effect:

$$\tilde{\Gamma}_{\sigma L}^{\parallel} = \Gamma_{\sigma L} \frac{\langle \sigma^z \rangle}{\langle S^z \rangle}. \tag{61}$$

When the static magnetic field is perpendicular to the crystal symmetry axis, the renormalization of the kinetic coefficients corresponding to the individual spin motion of the resonant and nonresonant components of the total magnetization gives

$$\begin{aligned} \Gamma_{ss}^{\bar{\lambda}\lambda} &= \pi \frac{\omega}{\omega_s} \bar{U}^2 T \left(\cot^2 \varphi + \frac{3}{4} \right), & \Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} &= \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{ss}^{\bar{\lambda}\lambda}, \\ \Gamma_{\sigma s}^{\bar{\lambda}\lambda} &= \frac{\pi}{4} \frac{\omega}{\omega_s} \bar{U}^2 T \frac{1}{\sin^2(\varphi/2)}, & \Gamma_{s\sigma}^{\bar{\lambda}\lambda} &= \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{\sigma s}^{\bar{\lambda}\lambda}, \\ & & \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} &= \frac{g_\perp}{2 g_\sigma \rho (T + T_{\rm W})}. \end{aligned} \tag{62}$$

The kinetic coefficients providing the coupling between the resonant and nonresonant parts of the total magnetization take the form

$$\begin{aligned} \Gamma_{ss}^{\lambda\lambda} &= \frac{\pi}{4} \bar{U}^2 T, & \Gamma_{\sigma\sigma}^{\lambda\lambda} &= \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{ss}^{\lambda\lambda}, \\ \Gamma_{\sigma s}^{\lambda\lambda} &= \frac{\pi}{4} \bar{U}^2 T \frac{1}{\cos^2(\varphi/2)}, & \Gamma_{s\sigma}^{\lambda\lambda} &= \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{\sigma s}^{\lambda\lambda}. \end{aligned} \tag{63}$$

The relaxation rates of the type $\Gamma_{\alpha\alpha'}^{\bar{\lambda}\lambda}$ logarithmically diverge at $T \to T_{\rm GK}$ similarly to the case of the parallel orientation of the static field to the *c*-axis:

$$\Gamma_{ss}^{\bar{\lambda}\lambda} = \Gamma_{\sigma s}^{\bar{\lambda}\lambda} = \pi \frac{\omega}{\omega_s} \frac{T}{\ln^2(T/T_{\rm GK})}, \qquad \Gamma_{\sigma\sigma}^{\bar{\lambda}\lambda} = \Gamma_{s\sigma}^{\bar{\lambda}\lambda} = \frac{\langle S^z \rangle}{\langle \sigma^z \rangle} \Gamma_{ss}^{\bar{\lambda}\lambda}. \tag{64}$$

Asymptotic expressions for the relaxation rates of the type $\Gamma_{\alpha\alpha'}^{\lambda\lambda}$, on the contrary, do not show any increase with lowering temperature and imitate the usual Korringa and Overhauser relaxation rates in the absence of the Kondo effect:

$$\Gamma_{ss}^{\lambda\lambda} = \Gamma_{\sigma s}^{\lambda\lambda} = \frac{\pi}{4}\frac{\omega}{\omega_s}\bar{U}^2 T, \qquad \Gamma_{\sigma\sigma}^{\lambda\lambda} = \Gamma_{s\sigma}^{\lambda\lambda} = \frac{\langle S^z\rangle}{\langle \sigma^z\rangle}\Gamma_{ss}^{\lambda\lambda}. \tag{65}$$

One can see that the coupling between the resonant and nonresonant parts of the total magnetization becomes non-essential due to the Kondo effect: $\Gamma_{\alpha\alpha'}^{\lambda\lambda} \ll \Gamma_{\alpha\alpha'}^{\bar{\lambda}\lambda}$ in contrast to the second order result (46) with $\Gamma_{\alpha\alpha'}^{\lambda\lambda} \sim \Gamma_{\alpha\alpha'}^{\bar{\lambda}\lambda}$. It allow us to consider the resonant and nonresonant parts separately as in the cases of an isotropic system and the parallel orientation of the static magnetic field to the *c*-axis.

The relaxation rate of the collective spin mode follows the equation (52) with renormalized relaxation rates $\Gamma^{(1)}$ and $\Gamma^{(2)}$:

$$\Gamma_{\text{coll}}^{\perp} = \sqrt{\Gamma^{(1)}\Gamma^{(2)}}, \quad \Gamma^{(1)} = \Gamma_{sL}^{\perp} + \Gamma_{\sigma L}\frac{\langle \sigma^z\rangle}{\langle S^z\rangle}, \quad \Gamma^{(2)} = \Gamma_{sL}^{\perp} + \frac{g_{\parallel}}{g_{\perp}}\tilde{\Gamma}_{\sigma L}^{\parallel} + \tilde{\Gamma}_{ss}^{\parallel}, \tag{66}$$

where $\tilde{\Gamma}_{\sigma L}^{\parallel}$ and $\tilde{\Gamma}_{ss}^{\parallel}$ are given by (58). At $T \to T_{\text{GK}}$ the effective Korringa relaxation rate $\tilde{\Gamma}_{ss}^{\parallel}$ is much smaller than the spin lattice relaxation rates Γ_{sL} and $\tilde{\Gamma}_{\sigma L}$, which makes it possible to expand the square root $\sqrt{\Gamma^{(2)}}$ up to the first order in $\tilde{\Gamma}_{ss}^{\parallel}$. The result is

$$\Gamma_{\text{coll}}^{\perp}(T \to T_{\text{GK}}) = \Gamma_{sL}^{\perp} + \tilde{\Gamma}_{\sigma L}^{\perp} + \tilde{\Gamma}_{ss}^{\perp}, \quad \tilde{\Gamma}_{\sigma L}^{\perp} = \Gamma_{\sigma L}\frac{\langle \sigma^z\rangle}{\langle S^z\rangle}, \quad \tilde{\Gamma}_{ss}^{\perp} = \frac{1}{2}\tilde{\Gamma}_{ss}^{\parallel} = \frac{\pi}{8}\bar{U}^4 T \ln^2(T/T_{\text{GK}}). \tag{67}$$

The ESR linewidth is seen to be narrowed even more than in the case of the parallel orientation of the static magnetic field to the *c*-axis: $\tilde{\Gamma}_{ss}^{\parallel}/\tilde{\Gamma}_{ss}^{\perp} = 2$, $\tilde{\Gamma}_{\sigma L}^{\parallel}/\tilde{\Gamma}_{\sigma L}^{\perp} = g_{\perp}/g_{\parallel} > 1$ and $\Gamma_{sL}^{\parallel}/\Gamma_{sL}^{\perp} \approx 2$ as estimated in the work [10]. Such a dependence on the orientation of the static magnetic field is likely due to the partial breaking of the bottleneck regime condition in the case of the parallel orientation, when the inequality $\omega_{\sigma} \gg \omega_s$ is fulfilled.

Now we consider the angular dependence of the ESR parameters to be affected by the renormalization. The renormalized relaxation rate is still independent of the orientation of the microwave magnetic field as well as in the absence of the Kondo effect. The angular dependence of the ESR intensity becomes more smooth due to the renormalization of the *g*-factor components $g_{\perp}$ and $g_{\parallel}$ in accordance with (56). The equation (53) converts to

$$I(\eta) = I(0)\left[1 - \sin^2\varphi \sin^2\eta\right], \tag{68}$$

where $\varphi(T)$ is defined by (54). The ratio $I(0)/I(\pi/2)$ is reduced to $1/\cos^2\varphi$ instead of the large values $g_{\perp}^2/g_{\parallel}^2 \approx 400$ and $g_{\perp}^2/g_{\parallel}^2 \approx 14$ in the cases of $YbRh_2Si_2$ and $YbIr_2Si_2$, respectively.

6. Conclusion

We investigated the spin dynamics in the heavy fermion compounds $YbRh_2Si_2$ and $YbIr_2Si_2$ taking into account the coupling between the resonant and nonresonant components of the total magnetization as well as the coupling between the conduction electrons and localized moments.

The calculations to the second order in the Kondo interaction show that the picture of spin kinetics is strongly dependent on the anisotropy of the Kondo interaction and the orientation of the static magnetic field. When a system is isotropic or the static magnetic field is parallel to the crystal symmetry axis *c*, the coupling between the resonant and nonresonant components is absent and their spin motions can be considered separately. In this case the relaxation rate of the collective spin mode is determined by the resonant terms of the total transverse dynamical susceptibility.

If the static magnetic field is perpendicular to the *c*-axis and the Kondo interaction is highly anisotropic the resonant and nonresonant components are coupled as strong as the subsystems of conduction electrons and localized moments. This results in significant corrections to the effective relaxation rate as compared with the case of the vanishing coupling between the resonant and nonresonant terms. The Kondo interaction does not make a direct contribution to the ESR linewidth likely to the case of an isotropic system (its Hamiltonian commutes with a transverse component of the total spin operator). Although the indirect contribution of the Kondo interaction via the relaxation to the lattice still takes place, it is well reduced by the small values of the spin lattice relaxation rates, so that the ESR linewidth is narrowed as against the result obtained for the case of the parallel orientation of the static magnetic field to the *c*-axis. However, these conclusions are only valid at sufficiently high temperatures, when the Kondo anomalies are not important.

The Kondo effect makes the picture of spin dynamics quite different. Upon lowering the temperature to T_{GK} the coupling between the resonant and nonresonant components of the total magnetization becomes negligible against the strong coupling between the conduction electron and localized moments. The spin motions corresponding to the resonant and nonresonant components can be considered separately again and the ESR linewidth is determined by the resonant terms for any configuration of the static and microwave magnetic fields. The great reduction of the effective relaxation rate at low temperatures is now due to the common energy scale T_{GK} which regulates the temperature dependence of different kinetic coefficients and leads to their mutual cancelation in the collective spin mode.

Another point to discuss is the variation of the ESR parameters with the orientation of the microwave magnetic field (angle η in fig. 1b). The relaxation rate of the collective spin mode does not depend on the angle η both in the absence and in the presence of the Kondo effect. Concerning the ESR intensity it varies with the orientation of the microwave field rather sharply at high temperatures $T \gg T_{GK}$ but the Kondo effect leads to more smooth dependence due to the renormalization of the *g*-factor components $g_{\perp}$ and $g_{\parallel}$. We should also note that the nonresonant terms do not seem to affect the ESR intensity, at least, so far as concerns its angular dependence.

In conclusion, our research shows that nonresonant terms of the total susceptibility may affect the ESR parameters at high temperatures, when the Kondo anomalies are not important, but at lower temperatures the Kondo effect makes their contribution non-essential as against the resonant ones.

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