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ИЗОТРОПНАЯ ФАЗА НЕМАТИКОВ В ПОРИСТОЙ СРЕДЕ

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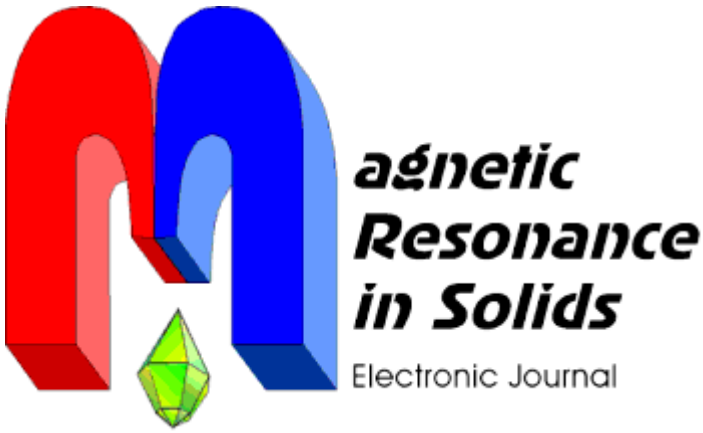

Volume **6**, *No.* **1**,
pages **95-102**, **2004**

http://mrsej.ksu.ru

ISOTROPIC PHASE OF NEMATICS IN POROUS MEDIA

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We study the effect of random porous matrices on the isotropic- nematic phase transition. Sufficiently close to the cleaning temperature, both random field and thermal fluctuations are important as disordering agents. A novel random field fixed point of renormalization group equation was found that controls the transition from isotropic to the replica symmetric phase. Explicit evaluation of the exponents in $d = 6 - \varepsilon$ dimensions yields to a dimensional reduction and three-exponent scaling.

Liquid crystalline ordering in a confined geometry has been the subject of considerable investigation during the past decade. The study of liquid crystals constrained to a random network of porous silica aerogel has been an area of current interest due to their importance in technological applications and from a fundamental point of view. Such liquid crystal porous matrix systems emerge in many natural and technological processes, giving rise to scientific activity. One of the fundamental questions is the effect of finite size and quenched disorder on the phase transitions. Liquid crystals exhibit a variety of experimentally accessible phase transitions involving orientational and translational ordering. Most of the studies are focused on the nematic-isotropic or nematic-smectic phase transitions. For example, the first has been investigated using various experimental techniques [1]. The main results could be summarized as follows: (a) the bulk isotropic-nematic (I-N) phase transition temperature is shifted down and the character of the transition changes; (b) even for above the bulk I-N phase transition temperature, there exists a weak residual nematic ordering; and (c) Monte Carlo simulations show that in some cases the nematic order is replaced by a quasi-long-range nematic phase.

Theoretical modeling of such phenomena is difficult. The porous matrix not only geometrically confines the liquid crystal, but also induces a random orienting field that fixes the direction of the order parameter near the surface of the matrix. Some experiments with liquid crystals in random porous media [2] showed that the random preferential orientation of the liquid crystal along the pore surface (whose normal changes direction randomly) profoundly influences the dynamics of an I-N phase transition in such a system, and fluctuations of the orientation order parameter relax at a much slower rate than in bulk liquid crystals. These experiments have stimulated theoretical work [3], and a random-field (RF) model for nematic liquid crystal has been proposed that qualitatively explains the glasslike behavior seen in experiments for liquid crystal-aerogel systems [2,4,5,6].

The nematic phase within the pores could be modeled as an Ising-like system with an imposed random field coupled directly to the orientational order parameter to account for the random confinement. Such a model uses a random uniaxial anisotropy on a spin system [3,7], including a symmetric coupling between the anisotropy vector and order parameter in order to account for the "up-down" nematic symmetry. This RF term in the Hamiltonian of the nematic liquid crystal is linear coupled to the order parameter. The strength of the random field in this model should directly depend on the anchoring strength of the molecules to the surface of the gel and indirectly on the porosity. This model may be described as an RF Ising model.

However, the experimentally obtained functional form for the scaled autocorrelation function is quite different from that obtained in a simulation of the RF Ising model. Really, the liquid crystal is a system with many degrees of freedom (the order parameter tensor has five independent components) and has a different symmetry from the Ising model.

The basic point in discussing the effect of RF on ordered nematic phases follows from the Imry-Ma argument [8,9], which suggests that this continuous symmetry system does not have nematic long-range order for dimensions less than four ($d < 4$). The possibility for the nematic phase to be replaced by a glassy state characterized by quasi-long-range order was discussed by Radzihovsky and Toner [10], and also predicted by numerical simulations [11], and by Feldman [12] using a renormalization group (RG) approach.

The theory [12] is the first one that extends beyond the mean field approximation for the low-temperature phase of disordered nematics. In this low-temperature phase, uniaxial nematics in random porous media can be mapped onto the RF $O(N)$ model. However, mapping becomes invalid near the phase transition to the isotropic phase. In this paper, we focus on the effects of quenched disorder that are introduced by the host silica aerogel at the high-temperature phase, i.e., above the I-N phase transition temperature. An appropriate model would require a full Landau-de Gennes type Hamiltonian incorporating a random orienting field. We carry out the mean field analysis and RG treatment as well.

The order parameter for a nematic liquid crystal is a three-dimensional symmetric traceless second rank tensor $Q_{\alpha\beta}$. The effective Landau-de Gennes free-energy functional appropriate to the RF nematic model near the I-N phase transition can be written as

$$F = \int d^d x \left\{ \frac{1}{2} r_0 \operatorname{Tr}\left(Q^2\right) + \frac{1}{2}\left(\nabla Q\right)^2 - \frac{1}{3} b \operatorname{Tr}\left(Q^3\right) + \frac{1}{4} c \left[\operatorname{Tr}\left(Q^2\right)\right]^2 - \operatorname{Tr}\left(h(x) Q(x)\right) \right\}, \tag{1}$$

where $r_0 = T - T_0$, T_0 is the second order transition temperature if $b = 0$ (bulk supercooled temperature limit), and b, c are temperature independent constants. The quenched RF $h_{\alpha\beta}(x)$ is a symmetric, traceless, Gaussian random tensor with vanishing quenched average $[h_{\alpha\beta}(x)]_{\alpha\upsilon} = 0$ and with variance [13]

$$[h_{\alpha\beta}(q) h_{\gamma\delta}(-q)]_{av} = h_0^2 \left(\frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}\right) - \frac{1}{n}\delta_{\alpha\beta}\delta_{\gamma\delta}\right), \tag{2}$$

n is the dimensionality of the tensor $h_{\alpha\beta}$.

Ground state configurations of the longitudinal component of the field $Q(x)$ (we consider here only the uniaxial nematic) are defined by the saddle-point equation

$$-\Delta Q + r_0 Q - b Q^2 + c Q^3 = h(x). \tag{3}$$

It will be useful to recall first what behavior is expected for a nematic placed in a non-random field, i.e., a homogeneous field in a uniform direction. The isotropic phase acquires some order and is transformed into a paranematic phase. The paranematic-nematic phase transition occurs at $r_{0c} = (2b^2/9c)\,(1 + h/2h_c)$. Here $h_c = (b^3/27c^2)$ is a uniform critical field that deter-

mines the nematic-paranematic critical point, $r_0^+(h_c) = b^2/3c$. For $h < h_c$ the paranematic supercooling temperature T_0 and the nematic overheating temperature T^* both have field dependence

$$\frac{h}{h_c} = 1 - 3\frac{T^+ - T}{T^+ - T_0} \pm 2\left(\frac{T^+ - T}{T^+ - T_0}\right)^{3/2}, \tag{4}$$

where (±) correspond to T_0 and T^*, respectively. All three temperatures T_0, T_c, and T^* merge at the nematic-paranematic critical point r_0^+ . For $h > h_c$, the order parameter Q will increase smoothly as temperature is decreased.

Apparently, the solutions of Eq. (3) with non-homogeneous $h(x)$ may essentially depend on a particular configuration of the quenched fields. The effect of RF is averaged over a length scale L, over which the orientation is correlated. The mean magnitude of the sum of the random fields is given by the sum of the squares of the random fields. Using the central limit theorem, the effective RF which couples to the local order parameter is approximately $h_0 L^{-d/2}$. Now, because the order parameter is changing on a length scale, the elastic energy term is of the form $(Q/L)^2$. Combining the ideas of Landau and those of Imry and Ma, it was shown, that for low order parameters $Q < ch_0/b^2$, the correlation length L is about a molecular length scale [7]. The free energy advantage is as though there were fixed nematic fields on the molecules, and it is negatively linear in Q . There is an energy cost in changing molecular orientation from point to point, but this is negligible because it is proportional to Q^2 . Thus, for the isotropic phase the effect in this mean field consideration is roughly the same whether the imposed field is random or fixed.

Let us estimate under which conditions random fields are relevant and are getting a dominant contribution for the ground state configurations. We divide the system into blocks of linear size L. As we have seen, the characteristic value of the RF in this block (averaged over realization) could be defined by $h = h_0 L^{-d/2}$. In the case when the fields can be considered as the dominant factor, the order parameter does not depend on the temperature and it happens for $h > \tau^{\beta\delta}$. Now it is easy to estimate the characteristic size of the block up to which the RF can dominate: $L < h_0^{2/d} \tau^{-2\beta\delta/d}$. On the other hand, the approximation we are using is correct only on length scales much larger than the fluctuation region $\xi \propto \tau^{-\nu}$. Thus, we have another bound for L: $L > \xi$. Therefore, the temperature region where RF effects cannot be ignored is,

$$|\tau| < \left(ch_0^2\right)^{1/(2\beta\delta - d\nu)} \equiv \tau_h . \tag{5}$$

Such a region of temperatures near T_c exists only if $2\beta\delta > d\nu$. This value of τ_h can be interpreted as the estimate for the temperature interval around T_c in which the order parameter configurations are essentially defined by the random fields.

In the mean field theory, using Landau critical exponents, the above nontrivial temperature interval τ_h exists only at dimensions $d < 6$ and equals $\tau_h = \left(ch_0^2\right)^{2/(6-d)}$. These simple arguments hold only in the approximation where critical fluctuations can be neglected. Thus, the temperature region τ_h where disorder induces a finite correlation length $\xi(h_0) \propto \left(ch_0^2\right)^{-1/(6-d)}$ is correct in this regime only.

It is easy to estimate the Ginzburg criterion of the applicability of this approximation. For our model (1), one can get $\tau_G \propto \max[b^{4/(6-d)}, c^{2/(4-d)}]$, and the above result is valid only for $\tau > \tau_G$. On the other hand, the Ginzburg temperature region is larger than the metastable interval of the first order I-N phase transition $\tau_G > b^2/c$. For weak RF such that $\tau_h < \tau < \tau_G$, critical exponents get renormalized by thermal fluctuations, and in the region $\tau < \tau_h$, RF fluctuations are important as well.

The following qualitative arguments may be constructed. Actually, multiple global solutions of the saddle point Eq. (3) can appear due to the double-well local potential. This potential has two local minima for $T_0 < T < T^*$ and for the values of the field $h < h_c$. At temperatures above T^*, the disordered local minima solution is unique. Just below T^*, however, multiple local minima solutions appear. The energy of the nematic solution is higher than typical energy of the disordered solutions. At further temperature lowering, the interaction of the local minima solutions is getting not small, and we may expect the nontrivial behavior. Like in spin-glasses [15], there is a large number of the disorder dependent local energy minima. In contrast to the usual spin-glass phase, these minima probably are separated by finite energy barriers. Therefore, it is possible to expect the existence of a finite temperature interval between isotropic and nematic phases where the glassy-type behavior occurs. In this state the standard nematic order parameter equals zero, $[\langle Q \rangle]_{av} = 0$, but the bilinear average $[\langle Q \rangle^2]_{av}$ is different from zero at all temperatures and plays the role of the order parameter of the nematic glass. At the same time, the application of external magnetic field H restores the long-range orientational order, and the magnetic field threshold is determined from the condition that the nematic coherence length $\xi_H \propto \left(\chi_a H^2\right)^{-1/2}$ is less than the disorder induced correlation length $\xi(h_0)$. Here, χ_a is the anisotropy of the diamagnetic susceptibility of the nematic.

In the glass-type phase, thermodynamics is defined by numerous disorder dependent local energy minima. In such a situation the perturbation theory and the usual RG approach in its traditional form that gives the correct result for the Hamiltonian with only one minimum cannot be used. The most developed technique in this case is the Parisi replica symmetry breaking method [16]. Using this technique, it has been proven that for the *N*-component ($N \gg 1$) spin systems with RF, the usual scaling replica-symmetry solution is unstable with respect to the replica symmetry breaking in the phase transition point. Moreover, it turns out that the spin-glass transition, which is believed to take place at replica symmetry breaking temperature, always precedes the low temperature phase and obeys the equation $\tau_{RSB} \propto (h_0^2)^{4/(6-d)}$ [17]. If we compare τ_{RSB} with a RF controlled temperature region τ_h, we see that $\tau_h > \tau_{RSB}$. It is not clear whether replica symmetry breaking occurs in the whole RF critical region τ_h, or is restricted to the much smaller temperature interval.

Now we consider the disordered I-N model, defined by Eq.(1) within the high-temperature, i.e., isotropic phase. A direct analysis of the Landau-de Gennes model written in terms of the order parameter is often superior in understanding the critical properties of the transition and the high-temperature phase. We assume the existence of rather strong fluctuations of the order parameter in the isotropic phase near the I-N transition, for which experimental evidence exists [18]. It is easy to show that all five independent tensor components are allowed to fluctuate in the isotropic phase in the same manner. This has considerably simplified the calculations in comparison with the nematic phase. On the other hand, the Landau-de Gennes Hamiltonian has cubic and quartic interaction terms, and one more is the RF term. Really, there are three length scales in the fluctuation theory. Namely, $\xi_c \propto c^{-1/(4-d)}$, $\xi_b \propto (b^2)^{-1/(6-d)}$ and $\xi_h \propto (ch_0^2)^{-1/(6-d)}$.

Let one remove the fast modes and rewrite the Hamiltonian in terms of the block order parameter, corresponding to the scale $L = al$. Here a is the ultraviolet cutoff, and $l > 1$. Then we make rescaling such a way that the Hamiltonian would restore its initial form with new constants $b(L)$, $c(L)$, and $h_0(L)$. Dimensional analysis provides estimations

$$b(L) = l^{(6-d)/2} b(a),\ c(L) = l^{4-d} c(a),\ h_0(L) = l h_0(a). \tag{6}$$

If one considers the combination $\Delta = ch_0^2$ as a new parameter, we immediately get

$$\Delta(L) = l^{6-d} \Delta(a). \tag{6a}$$

Iteration until $\Delta(L_0) = 1$ yields $L_0 = \xi_h$, i.e., the length scale beyond which the RF fluctuations are significant. The same arguments are true for the order parameter fluctuations coming from cubic term in (1). The quartic term is an irrelevant variable in the RG sense. Hence, the two length scales are important for I-N phase transition near $d = 6$. Thus we interpret this result physically by noting that sufficiently close to T_c, the dominant disordering agent is not the RF only, but the thermal fluctuations caused by cubic interaction also. Of course, eqs. (6) are not exact, and corrections to the renormalization due to the interaction are necessary. The leading corrections to equations (6) and (6a) are proportional to quadratic forms of Δ and b^2. Using the RG method for disordered systems, recursion relations are established for the parameters of the effective replica Hamiltonian. Then replica symmetry is assumed and the RG equations become simple functions of replica number. In that respect, the use of replica is a trick of diagram counting. One can generally establish identical RG equations directly by considering disorder correlation functions, a method which is usually called a replica symmetry perturbation theory. After standard RG transformations, the one-loop equations in differential form are the following

$$\frac{dr}{d\ln L} = (2-\eta)r - \frac{7}{6}b^2(1-2r) + 7\Delta(1-2r), \tag{7a}$$

$$\frac{d\ln b^2}{d\ln L} = \varepsilon - 3\eta - b^2 - 24\Delta, \tag{7b}$$

$$\frac{d\ln \Delta}{d\ln L} = \varepsilon - 3\eta + \frac{22}{3}b^2 - 26\Delta. \tag{7c}$$

Here we put $n = 3$ for a nematic liquid crystal.

The exponent η determines the behavior of the two-point correlation function $G(q)$, which is defined by means of the relation

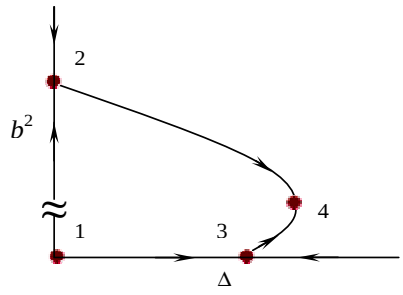
$$G(q) = [\langle Q(q)Q(-q)\rangle]_{av} - [\langle Q(q)\rangle\langle Q(-q)\rangle]_{av}. \tag{8}$$

At the critical point, $G(q)$ diverges as $q^{\eta-2}$, and to the lowest order in the perturbation expansion,

$$\eta = 7b^2/18. \tag{9}$$

Inserting this expression into (7), we find that the fixed points $\mu(b^2,\Delta)$ of the RG equations are given by $\mu_0(0,0)$, $\mu_b(6\varepsilon/13,0)$, $\mu_\Delta(0,\varepsilon/26)$, and $\mu^*(6\varepsilon/613,25\varepsilon/613)$. The RG flow diagram in the (b^2,Δ) plane is illustrated in figure.

In addition to the trivial Gaussian fixed point μ_0, these equations possess three nontrivial fixed points. The fixed point μ_b describes the critical behavior of the pure nematic and the coefficient r at this point is greater than zero. Thus



Phase diagram of the RG equations (7b) and (7c). The arrows show the direction of the renormalization group flows. Points 1, 2, 3, and 4 stand for the fixed points μ_0, μ_b, μ_Δ, and μ^*, respectively

fixed-point Hamiltonian has a minima at $Q = 0$ and at $Q \approx b/c$. The first-order transition occurs if the order parameter falls into the later deep minimum. It is likely that the μ_b fixed point corresponds to the critical fluctuations about the metastable minimum at $Q = 0$ [19]. The fixed point μ_Δ is exactly the RF Heisenberg fixed point for the five component $O(N)$ model and it describes the RF behavior at the isolated Landau point on the phase diagram, where $b = 0$ [20]. All the above fixed points are unstable.

The only stable fixed point is μ^*, that controls the behavior of the relevant parameters of the Hamiltonian below six dimensions, and corresponds to the replica symmetric phase with an infinite correlation length. The fixed point value of r is less than zero, i.e., the transition temperature is renormalized upward. Unfortunately, there is no unique energy extremum in this case. If the RG flows are smooth in the neighborhood of $r = 0$, as is usually assumed, then it should be reasonable to extrapolate the flows from $r \geq 0$ (where the RG equations are presumably valid) to the desired region near the fixed point.

Let us now determine the critical exponents associated with the fixed point μ^*. The correlation length exponent ν follows directly from Eq. (7a)

$$\nu^{-1} = 2 + \frac{35}{18} b^2 - 14\Delta . \tag{10}$$

To obtain an expression for the critical exponent for the susceptibility γ, we recall that under the RG iterations the two-point correlation function behaves like [21]

$$G = \exp(2\ell - \int_0^\ell \eta(\ell')d\ell')G_\ell, \quad \ell = \ln L .$$

Using the perturbation expansion for G_ℓ one can obtain

$$\gamma = 1 - \frac{7}{6} b^2 + 7\Delta . \tag{11}$$

The specific heat exponent α can be calculated from a singular part of the free energy

$$F_s \propto \int_0^{\ell^*} \ln(1 + \tilde{r}(\ell))e^{-d\ell} d\ell ,$$

where $\tilde{r}(\ell)$ is the coefficient of the Q^2 term in Hamiltonian averaged over the distribution of the RF, and $\ell^* = \ln \xi$. Evaluating the above integral to leading order we find $F_s \propto \tau^{2-\alpha}$, where

$$\alpha = \frac{6-d}{2} + \frac{7}{4} b^2 - 14\Delta . \tag{12}$$

Equations (9)-(12) yield the usual "thermodynamic" scaling law $\gamma = \nu(2 - \eta)$, and modified hyperscaling law $2 - \alpha = \nu(d - \theta)$ with the "violation of hyperscaling" exponent $\theta = 2 - \eta$. This result is valid at first order in $\varepsilon = 6 - d$.

In the presence of the RF, the quantity $[\langle Q(0)\rangle\langle Q(x)\rangle]_{av}$ is non-zero even in phases where $[\langle Q(x)\rangle]_{av}$ vanishes. There are therefore two distinct correlation functions to consider. The first is the analog of the usual connected correlation function G (8), and the second is the disconnected function and is specific for random systems. It measures the fluctuations in the local quenched order parameter

$$C_s(q) = [\langle Q(q)\rangle\langle Q(-q)\rangle]_{av} - [\langle Q(q)\rangle]_{av}[\langle Q(-q)\rangle]_{av} \tag{13}$$

and diverges at small q as $C_s \propto q^{\bar{\eta}-4}$.

To compute the exponents describing the behavior of the disconnected correlation function near T_c, we can write $C_s(q) = G^2(q)D(q)$ [22]. Here $D(q)$ is related to a dressed spectral function $h_0^2(q)$ of the RF fluctuations. If $h_0^2(q) \propto q^{-\lambda_\Delta}$ for $q\xi \gg 1$, then one obtains $C_s(q) \propto q^{2\eta-4-\lambda_\Delta}$ and $\tilde{\eta} = 2\eta - \lambda_\Delta$. Note that the choice $\lambda_\Delta = 0$ yields $\tilde{\eta} = 2\eta$, a value which is on the limit of the exact inequality $\tilde{\eta} \leq 2\eta$, due to [23]. Another relation was suggested by consideration of the RF contribution to the free energy in a correlation volume which scales as ξ^θ. In contrast, for the pure system the characteristic scale of variation of the effective free energy is simply set by the thermal fluctuations, i.e., $\propto T$. On the other hand, if the local order parameter was uncorrelated with the RF this would scale as $\xi^{d/2-\beta/\nu}$.

Here the factor $\xi^{d/2}$ is coming from the scaling of the total RF. The relation for β is easy to find from scaling the disconnected correlation function in a real space $2\beta = (d - 4 + \tilde{\eta})\nu$. Since the correlations could be included by additional factor $\xi^{\lambda_\Delta/2}$, corresponding to $h_0^2(q) \propto \xi^{\lambda_\Delta}$ for $q\xi \ll 1$, we expect that $\theta = 2 - \eta + \lambda_\Delta$. The case $\lambda_\Delta = 0$ yields $\theta = 2 - \eta$, which is on the limit of another inequality $\theta \geq 2 - \eta$ [24]. Thus for the violation of hyperscaling exponent we recover the result $\theta = 2 + \eta - \tilde{\eta}$ [14,17,24,25].

In our one-loop calculations, the critical exponent η is determined by the coupling b (9), and $\tilde{\eta} = 2\eta$. The Δ dependent term appears in $C_s(q)$ only in the two-loop diagrams, as does the cross term $b^2\Delta$. Using the diagrammatic expansion for $D(q)$ we find

$$D(q) = h_0^2(1 + \frac{7}{3}\Delta(b^2 - 6\Delta)\ln q), \tag{14}$$

and now the particular value $\lambda_\Delta = (7/3)\Delta(6\Delta - b^2)$ is non-zero to the second order in $\varepsilon = 6 - d$. All diagrams in (14) must be disconnected before averaging over the RF distribution.

The value for η is coming from the diagrams contributing to two-point correlation function $G(q)$. There are three types of terms in the perturbation expansion. The first and second terms are a double power in b^2 and Δ, respectively. The third term is a double product $b^2\Delta$ contribution. In contrast to the disconnected correlation function, all diagrams here are connected before configuration averaging. Note that not all of them are tree-like diagrams, as it is the case for the $O(N)$ model. Formally we can divide all these contributions and write $\eta = \eta_1(b^2, b^4) + \eta_2(\Delta^2, b^2\Delta)$. In the one-loop approximation, $\eta_2 = 0$ and η_1 is the critical exponent to linear order in ε (9). A straightforward evaluation of the RF depended diagrams leads to expression $\eta_2 = \lambda_\Delta$. This means that for the hyperscaling violation exponent we get $\theta = 2 - \eta_1$.

More generally, in the vicinity of the fixed point μ^* the random correlation function is proportional to c^{-1} for small c. Therefore, in the critical region one expects that the random correlation function will scale as $C_s(q, \xi, c) = c^{-1}e^{(2-\eta-\lambda_c)\ell}C_s(e^\ell q, e^{-\ell}\xi)$, where λ_c is the scaling exponent of the irrelevant parameter c. For $q = 0$ one has the behavior $C_s(0, \tau) \propto \tau^{-\tilde{\gamma}}$ with $\tilde{\gamma} = \nu(2 - \eta - \lambda_c)$. Using the relation $\tilde{\gamma} = \nu(4 - \tilde{\eta})$ that follows from the scaling at small q and $\tau = 0$ we can write $\lambda_c = \tilde{\eta} - \eta - 2$. We see that $\lambda_c = -\theta$. This result is quite obvious. Really, on the other hand, the perturbation expansion for free energy is a double power series in b^2, c, and h_0^2. The first terms in this series behave like $b^2h_0^2$ and ch_0^4, or for large h_0^2 they both are proportional to h_0^2 as well. Thus, for the free energy density we have $F(h_0^2, \tau) = \tau^{\nu d}f(h_0^2\tau^{-\varphi})$, where φ is the crossover exponent. If we conclude that f is a linear function of its argument for small τ, as it follows from the perturbation expansion, one can get $F(h_0^2, \tau) \propto \tau^{\nu(d-\varphi/\nu)}$, hence $\theta = \varphi/\nu$. The crossover exponent is related to the scaling of the RF near the fixed point μ^*: h_0^2 increases as $\exp(\ell\varphi/\nu)$. Writing the recursion relation for h_0^2 up to two-loop order, as we have done, Eq.(14), we again find $\varphi/\nu = 2 - \eta + \lambda_\Delta = 2 - \eta_1$.

All our results for critical exponents suggest that $\tilde{\eta} \neq 2\eta$, in agreement with the three-exponent scaling picture [24,25]. For example, the exponent scaling gives for the ratio $C_s(0)/G^2(0) \propto \xi^{\lambda_\Delta}$, which would diverge unless $2\eta = \tilde{\eta}$ is valid. However, this divergence is too weak to be detected, and thus this ratio may be concerned as a constant, and the concept of no self-averaging in RF systems is expected [26].

We have considered the effects of a RF (field conjugate to the order parameter) on an I-N phase transition using the $\varepsilon = 6 - d$ expansion method. We have found the novel RF fixed point that proceeds from the existence of two relevant variables in the RG approach, namely, RF and quartic interaction product, Δ, and the cubic interaction, b, which is special to a nematic liquid crystal. The first involves the effects of the RF, while the latter involves those of thermal disorder. These two agents of disorder give comparable contributions to the problem. In the pure nematic, when $h_0 = 0$, the zero cubic term means that the system is located at an isolated Landau point at the phase diagram. This point is unstable with respect to b. The interpretation of this instability depends on the existence of a stable μ_b fixed point. As was mentioned, this fixed point corresponds to critical fluctuations about the metastable minimum at $Q = 0$. When non-zero h_0 is switched on, Δ scales as $\exp(50\varepsilon\ell/13)$ near the pure fixed point. Then Δ is renormalized toward a fixed point μ^*, and all critical exponents are changed. As we believe, this fixed point governs the critical behavior at the transition from isotropic to the replica symmetric phase, that precedes the replica symmetry breaking phase. Such kind of two step scenario is likely to take place in the Ising spin glass in an external magnetic field [27]. The location of this nontrivial random fixed point on a phase diagram is quite close to the fixed point μ_Δ with zero cubic term (we may call this point a random isolated Landau point). This indicates that the critical behavior of the isotropic nematic in RF is like the behavior of the RF Heisenberg model for the five component order parameter. The independent calculation of the critical exponents shows that the dimensional reduction in the hyperscaling relations for the RF isotropic nematic contains the

shifted value $d-2-\eta+\tilde{\eta}$ instead of *d*. The so-called "three exponent scaling" appears in the second order in ε. The model for studing the replica symmetry breaking transition from the replica symmetric phase is clearly necessary to perform further investigations.

I would like to thank Professor Boris Kochelaev for stimulating my interest in the problem of disorder in the soft condensed matter systems, and for acquainting me with the mysterious liquid crystals.

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