THEORY OF SUPERCONDUCTOR - STRONG FERROMAGNET CONTACTS

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ТЕОРИЯ КОНТАКТОВ СВЕРХПРОВОДНИК-СИЛЬНЫЙ ФЕРРОМАГНЕТИК

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On the basis of microscopic approach we derive the Eilenberger-type equations of superconductivity for metals with exchange-split conduction band. The equations are valid for arbitrary band splitting and arbitrary spin-dependent electron mean free paths within the quasiclassic approximation. Next, we deduce general boundary conditions for the above equations. These boundary conditions take into account explicitly spin-dependence of F/S interface transparency. We apply our theory for the Andreev reflection at F/S interface and derive an expression for the Andreev conductance at zero bias. Based on experimental data and our calculations we give estimations of the conduction band spin polarization for series of ferromagnets in contact with superconductors. Next, we consider the superconducting proximity effect for a contact of a strong and clean enough ferromagnet with a dirty superconductor. Our calculations show that superconducting T_c of an F/S bilayer oscillates as a function of the F-layer thickness. At small enough superconducting layer thickness the re-entrant behavior of superconductivity is predicted. The theory gives also nonmonotonic dependence of the superconducting layer critical thickness on the spin-polarization of the ferromagnetic layer. These unconventional and distinctive features of the F/S proximity effect fit well experimental observations.

1. Introduction

At low temperatures, an electric current flows through a normal metal/superconductor (N/S) interface as a result of Andreev reflection [1]. An electron is reflected from the N/S interface as a hole into a subband with the opposite spin, and the formed Cooper pair moves through the superconductor transferring the charge 2*e*. The doubling of differential conductance of a pure N/S microcontact was demonstrated theoretically in [2] (BTK) based on the solution of the Bogoliubov equations. In Ref. [3] an attention was drawn to the fact that the Andreev reflection in ferromagnet/superconductor (F/S) contacts is suppressed as the spin polarization of the ferromagnet conduction band sets up. This is associated with the fact that the Andreev reflection efficiency decreases with diminishing the number of conducting channels in the minority spin-subband (the subband with lower value of the Fermi momentum). In Refs. [4-7], it was suggested that the suppression of Andreev reflection in F/S contacts can be used for determining the spin polarization of the conduction band of ferromagnets (Andreev spectroscopy of ferromagnets). Experimental data were interpreted making use of either general phenomenological considerations that the spin-polarized component of the normal current does not pass through the singlet superconductor [4-6], or the BTK equations semi-phenomenologically adapted to the F/S contacts [8,9]. More elaborated treatment had been proposed in Ref. [10] including the case of diffusive conductance in the vicinity of the contact. The BTK theory was generalized and applied to F/S point contacts in the theoretical works [11-13]. The expressions obtained for the Andreev conductance in the above works are not consistent with each other. Moreover, the results obtained in [11,12] do not reproduce the Andreev conductance at zero temperature, which follows from physical considerations and previous work, as we will show below. Number of experiments on Andreev spectroscopy of ferromagnets grows [8,9,14-20], what demands an adequate theoretical understanding and description.

Another motivation is the recent discussion of oscillatory phenomena in S/F contacts as a function of the ferromagnetic layer thickness [21,22]. Calculations of the local tunnelling density of states [21,22] and the superconducting transition temperature [22] do not predict oscillations if the ferromagnet is clean (with mean free path much longer than the pairing function oscillation length).

The paper summarizes our recent efforts to build a consistent quasiclassical theory of superconductor-ferromagnet contacts, as well as Andreev reflection and proximity effects for the heterogeneous structures with an interface of superconductor and strong ferromagnet [23-25]. We derive quasiclassical equations of superconductivity for metals with spin-split conduction band and deduce boundary conditions (BCs) for the quasiclassical Green's functions (GFs) at F/S interface. Next, we compute the Andreev conductance of an F/S point contact and give an estimate for the polarization of conduction bands of ferromagnetic metals from comparison with experiments on the Andreev spectroscopy. Finally, we analyze proximity effect in the superconductor - strong ferromagnet bilayer structure.

2. Equations of superconductivity and general boundary conditions

2.1. Eilenberger-type equations for a metal with exchange-split conduction band

We start from equations for equilibrium thermodynamic GFs in a matrix form [26], taking into account explicitly the spin splitting of the conduction band,

$$\left(i\varepsilon_n\tau_z + \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + \hat{\Delta} + \hat{\mu} - U - \hat{\Sigma}\right)\hat{G}(\mathbf{r},\mathbf{r}',\varepsilon_n) = \delta(\mathbf{r}-\mathbf{r}'). \tag{1}$$

Here, the Green function $\hat{G}$ and the self-energy part $\hat{\Sigma}$ are matrices of the form

$$\hat{G} = \begin{pmatrix} G_{\alpha\alpha} & F_{\alpha-\alpha} \\ -\overline{F}_{-\alpha\alpha} & \overline{G}_{-\alpha-\alpha}\end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} \Sigma_{\alpha\alpha} & \Sigma_{\alpha-\alpha} \\ -\overline{\Sigma}_{-\alpha\alpha} & \overline{\Sigma}_{-\alpha-\alpha}\end{pmatrix}. \tag{2}$$

In addition,

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0\end{pmatrix}, \quad \hat{\mu} = \frac{1}{2m}\begin{pmatrix} p_{F\uparrow}^2 & 0 \\ 0 & p_{F\downarrow}^2\end{pmatrix}, \tag{3}$$

τ_α are the Pauli matrices, α is the spin index, $\varepsilon_n = (2n+1)\pi T$ is the Matsubara frequency, Δ is the order parameter, p_α^F is the Fermi momentum, U is the electron interaction energy with the electric potential, $\mathbf{r} = (x, \rho)$, and $\rho = (y, z)$. Hereafter we use the unit system at which $\hbar = c = 1$, so that we do not distinguish momentum and wave number, for example. We assume that the F/S interface coincides with the plane $x = 0$. Making use the Fourier transformation with respect to the coordinate ($\rho - \rho'$) we obtain the equation for $\hat{G}(x,x') = \hat{G}(x,x',\rho_c,\mathbf{p}_\parallel,\varepsilon_n)$ $\mathbf{p}_\parallel$ is the projection of the momentum onto the contact plane, $\rho_c = (\rho + \rho')/2$ is the center-of-mass coordinate, in that follows the index "*c*" of ρ_c will be omitted for brevity):

$$\left(i\varepsilon_n\tau_z + \frac{1}{2m}\frac{\partial^2}{\partial x^2} + i\frac{\mathbf{v}_\parallel}{2}\frac{\partial}{\partial\rho} + \frac{\hat{p}_x^2}{2m} + \hat{\Delta} - U - \hat{\Sigma}\right)\hat{G}(x,x') = \delta(x-x'), \tag{4}$$

$$\frac{\hat{p}_x^2}{2m} = \hat{\mu} - \frac{\hat{p}_\parallel^2}{2m}.$$

For the $\hat{G}(x,x')$ function, we will use Zaitsev representation [27] taking into account in a systematic way the spin splitting of the conduction band. The quantities related to the metal on the left (right) side of the interface will be designated by indices 1 (2). For the sake of definiteness, we assume that the index 1 corresponds to the ferromagnet (F) and the index 2 – to the superconductor (S). Thus, for $x,x < 0'$,

$$\hat{G} = e^{i\hat{p}_{1x}x}\hat{G}_{11}e^{-i\hat{p}_{1x}x'} + e^{-i\hat{p}_{1x}x}\hat{G}_{22}e^{i\hat{p}_{1x}x'} + e^{i\hat{p}_{1x}x}\hat{G}_{12}e^{i\hat{p}_{1x}x'} + e^{-i\hat{p}_{1x}x}\hat{G}_{21}e^{-i\hat{p}_{1x}x'}. \tag{5}$$

Here, $\hat{p}_{1x} = \left[\hat{p}_{1F}^2 - \hat{p}_\parallel^2\right]^{1/2}$; for $x, x' > 0$, $\hat{p}_{1x}$ in Eq. (5) should be changed to $\hat{p}_{2x}$. Substituting Eq. (5) into Eq. (4) and neglecting the second derivative with respect to x we obtain equations for $\hat{G}_{kn}(x,x')$:

$$\left(i\varepsilon_n\tau_z - i(-1)^k v_{1x}\frac{\partial}{\partial x} + i\frac{\mathbf{v}_\parallel}{2}\frac{\partial}{\partial\rho} + \tilde{\tilde{\Delta}}_k - U - \tilde{\tilde{\Sigma}}_k\right)\hat{G}_{kn}(x,x') = 0, x \neq x'. \tag{6}$$

Here, $\tilde{\tilde{\Delta}}_k = e^{i(-1)^k\hat{p}_{1x}x}\hat{\Delta}e^{-i(-1)^k\hat{p}_{1x}x}$, and $\tilde{\tilde{\Sigma}}_k$ is determined in the same way. For $x,x' > 0$, v_{1x} in Eq. (5) should be replaced by v_{2x}. The equation conjugate to Eq. (6) is derived similarly. Let us pass now to the functions $\hat{g} = \hat{g}(x, x', \hat{p}_{jx})$ and $\widehat{G} = \widehat{G}(x, x', \hat{p}_{jx})$, which depend on the sign of the variable $\hat{p}_{jx}$ and are continuous at the point $x = x'$:

$$\hat{g} = \begin{cases} \hat{g}_> = 2i\sqrt{\hat{v}_{jx}}\hat{A}_1(x)\hat{G}_{11}(x,x')\hat{A}_1^*(x)\sqrt{\hat{v}_{jx}} - \mathrm{sgn}(x-x'), & \hat{p}_{jx} > 0; \\ \hat{g}_< = 2i\sqrt{\hat{v}_{jx}}\hat{A}_2(x)\hat{G}_{22}(x,x')\hat{A}_2^*(x)\sqrt{\hat{v}_{jx}} + \mathrm{sgn}(x-x'), & \hat{p}_{jx} < 0; \end{cases} \tag{7}$$

$$\widehat{G} = \begin{cases} \widehat{G}_> = 2i\sqrt{\hat{v}_{jx}}\hat{A}_1(x)\hat{G}_{12}(x,x')\hat{A}_2^*(x)\sqrt{\hat{v}_{jx}} - \mathrm{sgn}(x-x'), & \hat{p}_{jx} > 0; \\ \widehat{G}_< = 2i\sqrt{\hat{v}_{jx}}\hat{A}_2(x)\hat{G}_{21}(x,x')\hat{A}_1^*(x)\sqrt{\hat{v}_{jx}} + \mathrm{sgn}(x-x'), & \hat{p}_{jx} < 0. \end{cases} \tag{8}$$

In Eq. (5) $\hat{A}_k(x) = \exp\{-i(-1)^k(\hat{p}_{jx} - \hat{\tau}_x\hat{p}_{jx}\hat{\tau}_x)x/2\}$. Let us substitute Eqs. (7) and (8) into Eq. (6) and into the equation conjugate to Eq. (6). Finding the difference (for $n = k$) and the sum (for $n \neq k$) of the resulting equations, we obtain the quasiclassical equations of superconductivity in a metal with the spin-split conduction band

$$\begin{gathered} \mathrm{sgn}(\hat{p}_{xj})\frac{\partial}{\partial x}\hat{g} + \frac{1}{2}\mathbf{v}_\parallel\frac{\partial}{\partial\rho}(\hat{v}_{xj}^{-1}\hat{g} + \hat{g}\hat{v}_{xj}^{-1}) + [\hat{K},\hat{g}]_- = 0, \\ \mathrm{sgn}(\hat{p}_{xj})\frac{\partial}{\partial x}\widehat{G} + \frac{1}{2}\mathbf{v}_\parallel\frac{\partial}{\partial\rho}(\hat{v}_{xj}^{-1}\widehat{G} - \widehat{G}\hat{v}_{xj}^{-1}) + [\hat{K},\widehat{G}]_+ = 0, \\ \hat{K} = -i\hat{v}_{xj}^{-\frac{1}{2}}(i\varepsilon_n\tau_z + \hat{\Delta} - \hat{\Sigma})\hat{v}_{xj}^{-\frac{1}{2}} - i(\hat{p}_{xj} - \hat{\tau}_x\hat{p}_{xj}\hat{\tau}_x)/2, \\ [a,b]_\pm = ab \pm ba. \end{gathered} \tag{9}$$

The impurity self-energies in Eqs. (9) are

$$\hat{\Sigma}^F = -ic\,|\,u\,|^2\int\frac{d\mathbf{p}_\parallel}{(2\pi)^2}(\hat{v}_x)^{-\frac{1}{2}}\hat{g}^F(\hat{v}_x)^{-\frac{1}{2}}, \tag{10}$$

$$\hat{\Sigma}^S = -i\frac{1}{2\tau^S} < \hat{g}^S >, \quad \frac{1}{\tau^S} = c\,|\,u\,|^2\,\frac{mp^S}{\pi}, \tag{11}$$

where p^S is the Fermi momentum of a superconductor, *u* is the interaction potential of electrons and impurities, *c* is the concentration of impurities, τ^S is the mean free time of electrons in a superconductor, and brackets mean averaging over the solid angle: $<\ldots> = \oint d\Omega/4\pi$.

In the case of an F/S interface, as well as for an N/S interface [27], the system of quasiclassical equations arises. In addition to the functions $\hat{g}$, the functions $\widehat{G}$ appear, which describe waves reflected from the interface. The above Eilenberger-type equations for the metal with the exchange-field-split conduction band had been derived for the first time in [23,24]. They are valid for arbitrary band splitting and arbitrary spin-dependent electron mean free paths within the quasiclassic approximation. The system of Eqs. (9) must be supplemented with boundary conditions at F/S interface.

2.2. General boundary conditions for the Eilenberger equations

We characterize the F/S interface by the transmission coefficient, $\hat{D}$, and the reflection coefficient, $\hat{R} = 1 - \hat{D}$. In the paper, we do not consider interactions which lead to the spin flip of an electron upon its transmission through the interface. Therefore, matrices $\hat{D}$ and $\hat{R}$ have a diagonal form with respect to spin. They have the same matrix structure as $\hat{\mu}$ in Eq. (3). Taking into account the explicit form of GF given by Eq. (5), and matching the quasiclassical functions on both sides of the interface according to the procedure proposed by Zaitsev, we obtain BCs for the quasiclassical equations Eqs. (9). For $p_{\|} < \min(p_{\uparrow}^F, p_{\downarrow}^F, p^S)$ (here, $p_{\uparrow}^F$ and $p_{\downarrow}^F$ are the Fermi momenta of the spin-subbands of a ferromagnet) it is convenient to represent these conditions in the matrix form

$$\begin{pmatrix} \hat{a}^* & -\hat{b}^* \\ -\hat{b} & \hat{a} \end{pmatrix} \left(V_x^F\right)^{1/2} \begin{pmatrix} \hat{g}_>^F & \widehat{G}_>^F \\ \widehat{G}_<^F & \hat{g}_<^F \end{pmatrix} \left(V_x^F\right)^{-1/2} = \left(V_x^S\right)^{1/2} \begin{pmatrix} \hat{g}_>^S & \widehat{G}_>^S \\ \widehat{G}_<^S & \hat{g}_<^S \end{pmatrix} \left(V_x^S\right)^{-1/2} \begin{pmatrix} \hat{a}^* & \hat{b}^* \\ \hat{b} & \hat{a} \end{pmatrix}. \quad (12)$$

Here $\hat{a} = \hat{d}^{-1}$, $\hat{b} = \hat{r}\hat{d}^{-1}$, $\hat{r}$ and $\hat{d}$ are the scattering amplitudes at the F/S interface [27], and the matrix $\left(V_x^{F(S)}\right)^{1/2}$ is the result of the direct product of the unit matrix and $\hat{v}_x^{F(S)}$. Let us pass in Eq. (12) to the functions $\tilde{\hat{g}}$ and $\tilde{\widehat{G}}$ using the relations:

$$\begin{gathered} \tilde{\hat{g}}_>^F = e^{i\frac{\hat{\vartheta}_r}{2}} \hat{g}_>^F e^{-i\frac{\hat{\vartheta}_r}{2}}, \quad \tilde{\hat{g}}_<^F = e^{-i\frac{\hat{\vartheta}_r}{2}} \hat{g}_<^F e^{i\frac{\hat{\vartheta}_r}{2}}, \\ \tilde{\widehat{G}}_>^F = e^{i\frac{\hat{\vartheta}_r}{2}} \widehat{G}_>^F e^{i\frac{\hat{\vartheta}_r}{2}}, \quad \tilde{\widehat{G}}_<^F = e^{-i\frac{\hat{\vartheta}_r}{2}} \widehat{G}_<^F e^{-i\frac{\hat{\vartheta}_r}{2}}, \\ \tilde{\hat{g}}_>^S = e^{i\frac{\hat{\vartheta}_{rd}}{2}} \hat{g}_>^S e^{-i\frac{\hat{\vartheta}_{rd}}{2}} \quad , \tilde{\hat{g}}_<^S = e^{-i\frac{\hat{\vartheta}_{rd}}{2}} \hat{g}_<^S e^{i\frac{\hat{\vartheta}_{rd}}{2}}, \\ \tilde{\widehat{G}}_>^S = e^{i\frac{\hat{\vartheta}_{rd}}{2}} \widehat{G}_>^S e^{i\frac{\hat{\vartheta}_{rd}}{2}}, \quad \tilde{\widehat{G}}_<^S = e^{-i\frac{\hat{\vartheta}_{rd}}{2}} \widehat{G}_<^S e^{-i\frac{\hat{\vartheta}_{rd}}{2}}, \\ \hat{\vartheta}_{rd} = \hat{\vartheta}_r/2 - \hat{\vartheta}_d, \end{gathered} \quad (13)$$

where $\hat{\vartheta}_r$ and $\hat{\vartheta}_d$ are the scattering phases associated with the scattering amplitudes $\hat{r}$ and $\hat{d}$ at the F/S interface, respectively. Next we pass to the $\tilde{\hat{g}}_{s(a)}$ and $\tilde{\widehat{G}}_{s(a)}$ matrices, symmetric (s) and antisymmetric (a) with respect to the variable p_{jx}:

$$\tilde{\hat{g}}_{s(a)} = \frac{1}{2}\left[\tilde{\hat{g}}_> \pm \tilde{\hat{g}}_<\right], \quad \tilde{\widehat{G}}_{s(a)} = \frac{1}{2}\left[\tilde{\widehat{G}}_> \pm \tilde{\widehat{G}}_<\right]. \quad (14)$$

After this transformation the boundary conditions can be solved with respect to the $\tilde{\widehat{G}}_{s(a)}$ matrices and take the form:

$$\begin{gathered} \left(\tilde{\hat{g}}_a^S\right)_d = \left(\tilde{\hat{g}}_a^F\right)_d, \quad (\tilde{\widehat{G}}_a^S)_d = (\tilde{\widehat{G}}_a^F), \\ \left(\sqrt{R_\alpha} - \sqrt{R_{-\alpha}}\right)(\tilde{\widehat{G}}_a^+)_n = \alpha_3\left(\tilde{\hat{g}}_a^-\right)_n, \\ \left(\sqrt{R_\alpha} - \sqrt{R_{-\alpha}}\right)(\tilde{\widehat{G}}_a^-)_n = \alpha_4\left(\tilde{\hat{g}}_a^+\right)_n, \\ -\tilde{\widehat{G}}_s^- = \sqrt{R_\alpha}(\tilde{\hat{g}}_s^+)_d + \alpha_1\left(\tilde{\hat{g}}_s^+\right)_n, \\ -\tilde{\widehat{G}}_s^+ = 1\sqrt{R_\alpha}(\tilde{\hat{g}}_s^-)_d + \alpha_2\left(\tilde{\hat{g}}_s^-\right)_n, \end{gathered} \quad (15)$$

where $\tilde{\hat{g}}_{s(a)}^{\pm} = 1/2\left[\tilde{\hat{g}}_{s(a)}^S \pm \tilde{\hat{g}}_{s(a)}^F\right]$, the $\tilde{\widehat{G}}_{s(a)}^{\pm}$ functions are determined in the same way. Indices n and d denote the diagonal and off-diagonal parts of the matrices: $\hat{T}_{d(n)} = 1/2[\hat{T} \pm \tau_z \hat{T} \tau_z]$. The coefficients α_i equal

$$\begin{gathered} \alpha_{1(2)} = \frac{1 + \sqrt{R_\alpha R_{-\alpha}} \mp \sqrt{D_\alpha D_{-\alpha}}}{\sqrt{R_\alpha} + \sqrt{R_{-\alpha}}}, \\ \alpha_{3(4)} = 1 - \sqrt{R_\alpha R_{-\alpha}} \pm \sqrt{D_\alpha D_{-\alpha}}. \end{gathered} \quad (16)$$

If the interference of waves arriving from neighboring interfaces can be neglected, and $\tilde{\hat{g}}$ does not depend on ρ, the boundary condition containing only the function $\tilde{\hat{g}}_a$ can be obtained:

$$\begin{gathered} \tilde{\hat{g}}_a^+ \hat{b}_1 + \hat{b}_2 \tilde{\hat{g}}_a^+ + \tilde{\hat{g}}_a^- \hat{b}_3 + \hat{b}_4 \tilde{\hat{g}}_a^- = \hat{b}_3 - \hat{b}_4, \\ \tilde{\hat{g}}_a^- \hat{b}_1 + \hat{b}_2 \tilde{\hat{g}}_a^- + \tilde{\hat{g}}_a^+ \hat{b}_3 + \hat{b}_4 \tilde{\hat{g}}_a^+ = \hat{b}_1 - \hat{b}_2. \end{gathered} \quad (17)$$

The $\hat{b}_i$ matrices equal

$$\hat{b}_1 = \tilde{\tilde{G}}_s^+ \tilde{g}_s^- + \tilde{\tilde{G}}_s^- \tilde{g}_s^+, \hat{b}_2 = \tilde{g}_s^- \tilde{\tilde{G}}_s^+ + \tilde{g}_s^+ \tilde{\tilde{G}}_s^-,$$
$$\hat{b}_3 = \tilde{\tilde{G}}_s^+ \tilde{g}_s^+ + \tilde{\tilde{G}}_s^- \tilde{g}_s^-, \hat{b}_4 = \tilde{g}_s^- \tilde{\tilde{G}}_s^- + \tilde{g}_s^+ \tilde{\tilde{G}}_s^+. \tag{18}$$

In this case, the system of BCs consists of Eqs. (17) and the first equation of Eqs. (15). Note that only the diagonal part of the functions is continuous at the F/S interface. These BCs take into account explicitly the spin-dependence of F/S interface. Previously quoted boundary conditions for the F/S interface can be obtained (if they are not wrong) as approximations, or limiting cases of our general BCs, Eqs. (12)-(18).

3. Andreev conductance of an F/S point contact

3.1. Basic formulas for a point contact

We consider an orifice of the radius a in an impenetrable membrane as a model for the point contact. At zero temperature, the expression for Andreev conductance G_A can be written from the following physical considerations. Let us find the current in the ferromagnet at $p_\downarrow^F < p^S, p_\uparrow^F$. Then, in the case of specular reflection from the interface, $p_\parallel = p_\downarrow^F \sin\theta_\downarrow = p_\uparrow^F \sin\theta_\uparrow = p^S \sin\theta^S$. The incidence angles of electrons, which can be Andreev reflected from the interface in the spin-up subband, are determined from the relationship $p_\downarrow^F \sin\theta_\downarrow = p_\uparrow^F \sin\theta_\uparrow$ and depend only on the parameter $\delta = p_\downarrow^F / p_\uparrow^F$. Electrons incident at more slanted trajectories will undergo total internal reflection. The problem becomes equivalent to the problem of finding the conductance of a point contact of normal metals with different Fermi momenta (in this case, they are $p_\downarrow^F$ and $p_\uparrow^F$) when these metals are in direct contact. Using the known solution of this problem by Zaitsev (eq. (38') in [27]), we find

$$G_A(T=0) = G_\downarrow \frac{8\delta(2+\delta)}{3(1+\delta)^2}, \quad G_\downarrow = \frac{e^2 \left(p_\downarrow^F\right)^2 A}{4\pi^4}, \tag{19}$$

where A is the contact area. The equations for $G_A(T = 0)$ obtained in [11,12] do not coincide with this result. At $\delta = 1$ (non-magnetic metal with equal $p_\downarrow^F$ and $p_\uparrow^F$), the Andreev conductance equals the doubled Sharvin conductance, which corresponds to the doubling of conductance as a result of Andreev reflection [1]. Let us now find an expression for the Andreev conductance in the case of arbitrary transmission coefficients. We start with equation for the electric current I in the linear approximation with respect to the electric field $\mathbf{E} = (E_x, 0, 0)$. The current is calculated on the ferromagnet side at $x \to 0$:

$$I_x = \frac{e^2}{2m^2} \lim_{\mathbf{r}\to\mathbf{r}'} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) \mathrm{Tr} \left\{ \tau_z \int_{-\infty}^{\infty} d\varepsilon \frac{1}{4\pi \cosh^2(\varepsilon/2T)} \right.$$
$$\left. \times \int d\mathbf{r}_1 G^R(\varepsilon, \mathbf{r}, \mathbf{r}_1) E_x(\mathbf{r}_1) \tau_z \frac{\partial}{\partial x_1} G^A(\varepsilon, \mathbf{r}_1, \mathbf{r}') \right\}. \tag{20}$$

Here, $G^{R(A)}$ is the retarded (advanced) GF, which is obtained from the temperature GFs (Eqs. (1) to (5) by substituting $\varepsilon \pm i\delta$ for $i\varepsilon_n$. Let us substitute representations given by Eqs. (5), (7) and (8) into Eq. (20). After performing the Fourier transformation with respect to the $\rho - \rho'$ coordinate, we obtain the ballistic conductance $G_{F/S}$ of an F/S point contact:

$$G_{F/S} = \frac{Ae^2}{16\pi} \mathrm{Tr} \left\{ \tau_z \int_{-\infty}^{\infty} d\varepsilon \frac{1}{\cosh^2(\varepsilon/2T)} \int \frac{d\mathbf{p}_\parallel}{(2\pi)^2} \right.$$
$$\left. \times \left[1 - \hat{g}_s^R \tau_z \hat{g}_s^A - \hat{g}_a^R \tau_z \hat{g}_a^A + \hat{G}_s^R \tau_z \hat{G}_s^A - \hat{G}_a^R \tau_z \hat{G}_a^A \right] \right\}. \tag{21}$$

Now, we must solve the first of Eqs. (9) with the BCs given by Eq. (17). When $\hat{g}$ is independent of ρ the solution to Eq. (9) takes the form

$$\hat{g}_j = e^{-sgn(\hat{p}_{jx})\hat{K}x} \hat{C}_j(\mathbf{p}_{Fj}) e^{sgn(\hat{p}_{jx})\hat{K}x} + \hat{C}_j. \tag{22}$$

Matrices $\hat{C}_j$ represent the values of GF $\hat{g}_j$ at large distances from the F/S interface:

$$\hat{C}_2 = \begin{pmatrix} g & f \\ -f^+ & -g \end{pmatrix} = 1\sqrt{\varepsilon_n^2 + |\Delta|^2} \begin{pmatrix} \varepsilon_n & -i\Delta \\ i\Delta^* & -\varepsilon_n \end{pmatrix}. \tag{23}$$

Passing in Eq. (22) to functions $\hat{g}_s^\pm$, substituting them into the system of BCs given by Eqs. (17), and solving them in the linear approximation with respect to $\hat{C}_a^\pm = 1/2[\hat{C}_a^S \pm \hat{C}_a^F]$ we find the Andreev conductance G_A of the F/S point contact:

$$G_A = G_{F/S}(V=0) = \frac{Ae^2}{4\pi}\int_0^{\Delta} d\varepsilon \frac{1}{\cosh^2(\varepsilon/2T)} \times \int \frac{d\mathbf{p}_\parallel}{(2\pi)^2} \frac{4|\Delta|^2 D_\downarrow D_\uparrow}{(1+\sqrt{R_\downarrow R_\uparrow})^2|\Delta|^2 - 4\sqrt{R_\downarrow R_\uparrow}\varepsilon^2}. \tag{24}$$

It depends on the relationship between the Fermi momenta $p_\downarrow^F$, $p_\uparrow^F$, and p^S. Thus, at $p_\downarrow^F < p^S < p_\uparrow^F$, the expression for G_A takes the form:

$$G_A = \frac{Ae^2(p_\downarrow^F)|\Delta|}{4\pi^2 T}\int_0^1 dx \frac{1}{\cosh^2\left(\dfrac{|\Delta|}{2T}x\right)} \times\int_0^{\pi/2} d\theta_\downarrow \frac{D_\downarrow D_\uparrow \sin(2\theta_\downarrow)}{(1+\sqrt{R_\downarrow R_\uparrow})^2 - 4\sqrt{R_\downarrow R_\uparrow}x^2}. \tag{25}$$

In the case of a nonmagnetic metal, where $D_\downarrow = D_\uparrow$, the expression for the Andreev conductance obtained in Ref. [27] follows from Eq. (25).

3.2. Discussion of experiments on the Andreev spectroscopy

The ratio of G_A to $G_{F/N}$, where $G_{F/N}$ is the conductance of an F/S contact in the normal state, is given in [4-8]. In our approach the latter quantity is

$$G_{F/N}(V=0) = \frac{Ae^2\left(p_\downarrow^F\right)^2}{8\pi^2}\int_0^{\pi/2} d\theta_\downarrow \sin(2\theta_\downarrow)D_\downarrow + \frac{Ae^2(p^S)}{8\pi^2}\int_0^{\pi/2} d\theta_N \sin(2\theta_N)D_\downarrow. \tag{26}$$

Eqs. (24)–(26) are valid for arbitrary transmission coefficients D_a. For particular calculations we use the model expressions for the transmission coefficients corresponding to the direct contact between S and F metals:

$$D_\uparrow = \frac{4p_{x\uparrow}p_x^S}{(p_{x\uparrow}+p_x^S)^2}, \quad D_\downarrow = \frac{4p_{x\downarrow}p_x^S}{(p_{x\downarrow}+p_x^S)^2}. \tag{27}$$

With these transmission coefficients, $G_A(T=0)$ and $G_{F/N}$ can be calculated analytically:

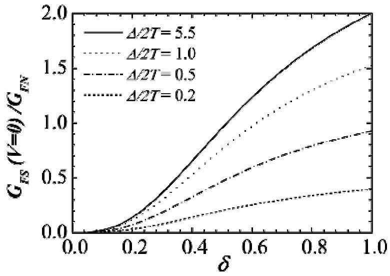

Fig. 1. Dependence of the normalized Andreev conductance on the ratio δ of the Fermi momenta of spin subbands of the ferromagnet's conduction band ($\delta = p_\downarrow^F/p_\uparrow^F$)

$$G_{F/N} = \frac{Ae^2\left(p^S\right)^2}{6\pi^2}\left(\frac{\delta_\uparrow^N(1+\delta_\uparrow^N)}{(1+\delta_\uparrow^N)^2} + \frac{(\delta_\downarrow^N)^3(2+\delta_\downarrow^N)}{(1+\delta_\downarrow^N)^2}\right), \tag{28}$$

for $G_A(T=0)$ the expression (19) is obtained. Here, $\delta_\uparrow^N = p^S/p_\uparrow^F$ and $\delta_\downarrow^N = p_\downarrow^F/p^S$. From Eqs. (19) and (28) it follows that the Andreev conductance at $\delta < 0.26$ becomes smaller than the conductance of the contact in the normal state. Dependence of the ratio $G_A(T=0)/G_{F/N}$ on the parameter δ is given for various temperatures in Fig. 1. The ratio $\Delta/2T = 5.5$ corresponds to the experimental conditions [5] ($T = 1.6$ K, $\Delta_{Nb} = 1.5$ meV). In order to interpret universally the experimental data obtained in [5] for a series of ferromagnetic materials in contact with superconducting Nb, we fixed the Fermi momentum of the superconducting metal by the equation $\left(p^S\right)^2 = \frac{1}{2}\left[\left(p_\uparrow^F\right)^2 + \left(p_\downarrow^F\right)^2\right] =$ const. Now, the values of δ (abscissa) can be estimated by the value of the reduced conductance at zero voltage across the contact (ordinate). Emphasize that in this calculation we assumed absence of an oxide or similar barrier at the F/S interface ($Z_{BTK} = 0$). The estimated results for δ are given in the Table.

Table

Material under study [5]	δ	P_c (%)	P_c (%) [5]
NiFe	0.64	42	37±5.0
Co	0.55	52	42±2.0
NiMnSb	0.48	63	58±2.3
LMSO	0.31	83	78±4.0
CrO2	0.18	94	90±3.6

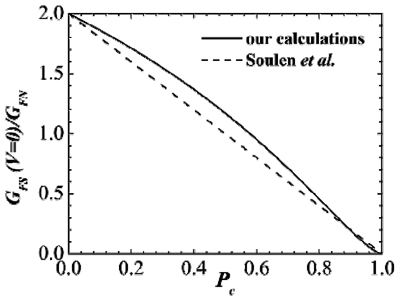

Fig. 2. Dependence of the normalized Andreev conductance on the contact polarization P_c

Note that our values δ(Ni) = 0.64 (from data of Ref. [6]) and δ(Co) = 0.55 obtained from the Andreev spectroscopy turned out to be close to the upper estimates for δ(Ni) = 0.64 and δ(Co) = 0.57, which we obtained in Ref. [28] from the data on the giant magnetoresistance in magnetic point contacts [29].

Let us now compare our results with the original estimates of polarization obtained in [5] (see the last column of the Table). The authors argue that the normalized conductance measured in their work depends on polarization as $G_{F/S}/G_n = 2(1-P_I)$ (Eqs. (4)-(6) in [5]), where $P_I = (I_\uparrow - I_\downarrow)/(I_\uparrow + I_\downarrow)$ and $G_n \simeq G_{F/N}$ is the conductance at high voltages across the contact ($eV \gg \Delta$). In the course of discussion, the authors identified the current polarization P_I with the contact polarization $P_c = (N_\uparrow v_\uparrow^F - N_\downarrow v_\downarrow^F)/(N_\uparrow v_\uparrow^F + N_\downarrow v_\downarrow^F) = (1-\delta^2)/(1+\delta^2)$, where N_α and v_α are the density of states and the Fermi velocity in the α-spin subband of the ferromagnet, respectively. This identification is not quite correct, because it implicitly assumes independence of the total current $I_\uparrow + I_\downarrow$ through the contact in the normal phase from the spin polarization of the ferromagnet. It is evident from Eq. (28) that $G_{F/N}$ essentially depends on δ. As a result, the reduced conductance $G_{F/N}(V=0)/G_{F/N}$ is the nonlinear function of the contact polarization P_c (Fig. 2). It is seen from Fig.2 that identification of P_I with P_c leads to a systematic underestimation of the P_c values extracted from experiment (compare the third and the fourth columns of the Table). Note here, that numerical calculations of the conductance at zero voltage performed in [13] (see Fig. 4 of that work for $Z = 0$, $T/T_c = 0.2$) fit well the linear dependence on the contact polarization proposed in [5] (dash line in our Fig. 2). From this observation it follows that calculations made in [13] also give underestimated values of the contact polarization taken from the conductance at zero voltage. Our theory allows to estimate the polarization parameter δ of the ferromagnet's conduction band, through which the polarization of the density of states P_{DOS}, the tunnelling polarization P_T, and the contact polarization P_c are expressed. Our analysis of experiments on Andreev spectroscopy leads to values of P_c that systematically higher than those estimated previously.

4. The superconductor - strong ferromagnet proximity effect

The ferromagnet-superconductor contacts are interesting not only as a tool to measure conduction band polarization of ferromagnets, but also as a unique combination of materials to build π-contacts [30] and superconducting logic circuits [31,32]. The physics behind the π-contacts is the unconventional Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) pairing in ferromagnetic superconductors [33,34], which manifests itself in F/S contacts by oscillations of the superconducting transition temperature [35,36], tunnelling density of states [37], Josephson current [38-41] as a function of F-layer thickness or temperature. The realization of the LOFF-like pairing in weak ferromagnets (ferromagnetic alloys having low Curie temperatures ~100 K) is certainly proved [32,37-39,41]. The case of the contacts of superconductors with strong ferromagnets, like Fe, Co, Ni, is still questionable. In fact, there are calculations of the local tunnelling density of states [21,22] and the superconducting transition temperature [22], which do not predict oscillations of the above mentioned quantities, if the ferromagnet is clean (with mean free path much longer than the pairing function oscillation length). Direct experimental verification is very difficult, because one needs ultra-thin, few monolayers thick, ferromagnetic films of excellent quality. In the section below we show, that in the case of clean enough ferromagnets the critical temperature still oscillates as a function of the F-layer thickness.

4.1. Boundary conditions for the "dirty" superconductor - strong ferromagnet bilayer

To find quasiclassic Green functions at the F/S interface one needs to solve the Eilenberger-type equations (9) for the every metal in a contact. Near the transition temperature the equations can be linearized with respect to the anomalous Green function $\hat{f}^{S(F)}$:

$$\hat{g}^{S(F)} = \frac{\varepsilon_n}{|\varepsilon_n|}\tau_z + \hat{f}^{S(F)}, \quad \hat{f}^{S(F)} = \begin{pmatrix} 0 & f_{\uparrow\downarrow}^{S(F)} \\ -\overline{f}_{\downarrow\uparrow}^{S(F)} & 0 \end{pmatrix}. \tag{29}$$

Then, the first of equations in (9), applied to the superconductor homogeneous in the plane of the contact, reads

$$\begin{gathered} l_x^2 \frac{\partial^2}{\partial x^2}\hat{f}_s^S - (\lambda^S)^2 \hat{f}_s^S + \lambda^S < \hat{f}_s^S >= 2i\lambda^S\tau^S\hat{\Delta}, \\ l_x \frac{\partial \hat{f}_s^S}{\partial x} = -\lambda^S \operatorname{sgn}(\varepsilon_n)\tau_z \hat{f}_a^S, \end{gathered} \tag{30}$$

where $l_x^S = |v_x^S|\tau^S$ is the component of the mean free path along the normal to the contact plane. Analogously, we deduce the equations to find GF $\hat{f}_s^F$ and $\hat{f}_a^F$ for the ferromagnet:

$$\left(\frac{2l_{x\uparrow}l_{x\downarrow}}{l_{x\uparrow}+l_{x\downarrow}}\right)^2\frac{\partial^2}{\partial x^2}\hat{f}_s^F-(\lambda^F)^2\hat{f}_s^F=-\frac{2\sqrt{v_{x\uparrow}v_{x\downarrow}}}{l_{x\uparrow}+l_{x\downarrow}}\tau_\uparrow\tau_\downarrow\lambda^F\left\langle\frac{1}{\tau_{\uparrow\downarrow}}\hat{f}_s^F\right\rangle,$$
$$\hat{f}_a^F=-\operatorname{sgn}(\varepsilon_n)\frac{2l_{x\uparrow}l_{x\downarrow}\lambda^F}{(l_{x\uparrow}+l_{x\downarrow})}\tau_z\frac{\partial}{\partial x}\hat{f}_s^F, \tag{31}$$

where

$$\lambda^F=1+\frac{2\tau_\uparrow\tau_\downarrow(v_{x\uparrow}+v_{x\downarrow})}{l_{x\uparrow}+l_{x\downarrow}}\mid\varepsilon_n\mid-i\frac{2l_{x\uparrow}l_{x\downarrow}}{l_{x\uparrow}+l_{x\downarrow}}(p_{x\uparrow}-p_{x\downarrow})\operatorname{sgn}(\varepsilon_n),$$
$$<\frac{1}{\tau_{\uparrow\downarrow}}\hat{f}_s^F>=c\mid u\mid^2\int\frac{d\mathbf{p}_\parallel}{(2\pi)^2}\frac{1}{\sqrt{v_{x\uparrow}v_{x\downarrow}}}\hat{f}_s^F. \tag{32}$$

In the above $l_{x\alpha}=\mid v_{x\alpha}^F\mid\tau_\alpha^F$, τ_α^F is the mean free path in the α-th spin-subband of the ferromagnet. In the second line of (32) the angular integration is constrained [42] to fulfil the specular scattering condition:

$$p_\parallel=p_\downarrow^F\sin\theta_\downarrow=p_\uparrow^F\sin\theta_\uparrow=p^S\sin\theta^S. \tag{33}$$

In Eqs. (31)-(33) and hereafter $p_\uparrow^F$ and $p_\downarrow^F$ are the Fermi momenta of the spin-subbands of the ferromagnet.

Solution of Eq. (30) with the boundary condition $\hat{f}_a^S(x\to\infty)=0$ reads:

$$\hat{f}_s^S(x)=\operatorname{sgn}(\varepsilon_n)\tau_z\hat{f}_a^S(x)$$
$$+\frac{1}{l_x^S}\int_x^\infty d\xi e^{-\frac{\lambda^S(\xi-x)}{l_x^S}}[<\hat{f}_s^S(\xi)>-2i\tau^S\hat{\Delta}(\xi)]. \tag{34}$$

In the equation (34) the integrand in the square parentheses has the spatial range $\xi_T^S=(D^S/2\pi T)^{1/2}\gg l^S$, where $D^S=v^Sl^S/3$ is the diffusion coefficient of electrons in a superconductor. Expanding the slow varying function around the point $\xi=x$ and taking it out of the integral we obtain $\hat{f}_s^S(x)$:

$$\hat{f}_s^S(x)=\operatorname{sgn}(\varepsilon_n)\tau_z\hat{f}_a^S(x)+\frac{1}{\lambda^S}\left(1+l_x^S\lambda^S\frac{d}{dx}\right)<\hat{f}_s^S(x)>. \tag{35}$$

Solution for the ferromagnet is sought in the form:

$$\hat{f}_s^F(x)=C^F(\theta_\downarrow)\cosh\kappa^F(\theta_\downarrow)(x+d^F), \tag{36}$$

where

$$\kappa^F(\theta_\downarrow)=\kappa_1^F(\theta_\downarrow)+i\operatorname{sgn}(\varepsilon_n)\kappa_2^F(\theta_\downarrow),$$
$$\kappa_1^F(\theta_\downarrow)=(1-\eta_1)\frac{l_{x\uparrow}+l_{x\downarrow}}{2l_{x\uparrow}l_{x\downarrow}}+\frac{v_{x\uparrow}+v_{x\downarrow}}{2v_{x\uparrow}v_{x\downarrow}}\mid\varepsilon_n\mid,$$
$$\kappa_2^F(\theta_\downarrow)=\mid p_{x\uparrow}-p_{x\downarrow}\mid+\eta_2\frac{l_\uparrow^F+l_\downarrow^F}{l_\uparrow^Fl_\downarrow^F}. \tag{37}$$

In the equations (37), the quantities η_1 and η_2 do not depend on the angles $\theta_\downarrow$ and $\theta_\uparrow$. As in the above Sections we assume $p_\uparrow^F>p_\downarrow^F$. Substituting the solution (36) into the equation (31) one obtains the integral equation to find η_1 and η_2:

$$\frac{2l_\uparrow}{p_\downarrow^Fp_\uparrow^F}\int\frac{d\mathbf{p}_\parallel}{2\pi}\frac{\lambda^F(l_{x\uparrow}+l_{x\downarrow})}{(\lambda^F)^2(l_{x\uparrow}+l_{x\downarrow})^2-(2l_{x\uparrow}l_{x\downarrow})^2(\kappa^F)^2}=1. \tag{38}$$

For the strong ferromagnet the solution of the above equation is (the relative accuracy is $\left(p_\uparrow^Fl_\uparrow^F\right)^{-1}\ll 1$):

$$\eta_1=\frac{l_\uparrow}{p_\downarrow^Fp_\uparrow^F}\int\frac{d\mathbf{p}_\parallel}{2\pi}\frac{1}{l_{x\uparrow}+l_{x\downarrow}},\eta_2=0. \tag{39}$$

Satisfying the boundary conditions (17) we obtain $\hat{f}_a^S(0)$ and $\hat{f}_a^F(0)$:

$$\hat{f}_a^S(0) = -\operatorname{sgn}(\varepsilon_n)\tau_z \frac{B}{\lambda^S}\left(1 + \frac{l_x^S}{\lambda^S}\frac{d}{dx}\right) < \hat{f}_s^S(x) >,$$
$$\hat{f}_a^F(0) = -2\operatorname{sgn}(\varepsilon_n)\tau_z \frac{\sqrt{D_\uparrow D_\downarrow}}{\Gamma\lambda^S}\left(1 + \frac{l_x^S}{\lambda^S}\frac{d}{dx}\right) < \hat{f}_s^S(x) >, \quad (40)$$

where

$$B = \frac{\Gamma^S}{\Gamma},$$
$$\Gamma^S = D_\uparrow + D_\downarrow + (\sqrt{R_\uparrow} - \sqrt{R_\downarrow})^2 \nu^F,$$
$$\Gamma = 2\left[1 + \sqrt{R_\uparrow R_\downarrow} + (1 - \sqrt{R_\uparrow R_\downarrow})\nu^F\right], \quad (41)$$
$$\nu^F = \frac{\lambda^F(l_{x\uparrow} + l_{x\downarrow})}{2 l_{x\uparrow} l_{x\downarrow}\kappa^F \tanh(\kappa^F d^F)}.$$

In deriving the above equations we neglected spin-dependence of the phases of the scattering amplitudes in the general boundary conditions Eqs. (17).

To formulate particular BC for the contact of a strong ferromagnet with a dirty superconductor we use the Ansatz proposed in Ref. [43]: at distances of the order of the mean free path $l_\downarrow^F / l_\uparrow^F = 2.5,$ in a superconductor, when the terms proportional to $\xi^S / \xi_{BCS}^S = 0.25$ and $\xi^S = (D^S / 2\pi T_{c0})^{1/2}$ can be neglected, one may write down

$$\left\langle \cos(\theta^S)\,\hat{f}^S \right\rangle = \frac{1}{2}\int_0^{\pi/2} d\theta^S \sin(2\theta^S)\,\hat{f}_a^S = \hat{C}^S, \quad (42)$$

where T_c is constant. This constant can be found substituting into Eq. (42) the antisymmetric combination

$$\hat{f}_a^S = -\hat{\tau}_z \operatorname{sgn}(\varepsilon_n) l_x^S \frac{d}{d\,x} < \hat{f}_s^S >,$$

which corresponds to the solution of the Usadel equation [44] for the "dirty" superconductor far away from the F/S interface. The result is:

$$\hat{C}^S = -\hat{\tau}_z \operatorname{sgn}(\varepsilon_n)\frac{1}{3} l^S \frac{d}{dx} < \hat{f}_s^S > . \quad (43)$$

Now, we calculate the same constant with the use of the function $\hat{f}_a^S(0)$ taken from the equation (40) above, and obtain the boundary condition for the averaged over the solid angle GF, $\hat{F}_s^S(x) = < \hat{f}_s^S >$:

$$l^S \frac{d}{dx}\hat{F}_s^S(x) = \gamma \hat{F}_s^S(x), \quad \gamma = \frac{\gamma_1}{1-\gamma_2},$$
$$\gamma_1 = \frac{3}{2}\int_0^{\varphi} d\theta^S \sin(2\theta^S) B, \quad \gamma_2 = \frac{3}{2}\int_0^{\varphi} d\theta^S \cos(\theta^S)\sin(2\theta^S) B. \quad (44)$$

The upper limit in Eq. (44) depends on the relation between the Fermi momenta of contacting metals, and is determined from conservation of the parallel component of the transferred momentum (33). If the Fermi momentum p^S is the smallest of three, $\varphi = \pi/2$. The quantity B is determined in Eq. (41), it is a function of the angles θ^S and $\theta_\downarrow$, which obey the scattering specularity condition (33). The boundary condition (44) is valid for the dirty superconductor - strong ferromagnet interface at arbitrary transparency.

4.2. Critical temperature of F/S bilayer

To find the superconducting transition temperature T_c of the bilayer we solve the linearized Usadel equation at temperatures close to T_c:

$$D^S \frac{d^2}{dx^2}\hat{F}_s^S - 2\,|\,\varepsilon_n\,|\,\hat{F}_s^S = 2i\hat{\Delta}, \quad (45)$$

and satisfy with this solution the boundary condition (44). The problem is easily solved in the single-mode approximation [45], which is valid at intermediate supessions of T_c against unperturbed transition temperature of the isolated superconducting film, T_{c0}:

$$\hat{F}_s^S = -\frac{2i\hat{\Delta}}{|\,\varepsilon_n\,| + D(\kappa^S)^2}, \quad \hat{\Delta} = \hat{\Delta}_0 \cos[\kappa^S(x - d^S)], \quad (46)$$

where κ^S is determined from BC (44):

$$l^S \kappa^S \tan(\kappa^S d^S) = \gamma. \qquad (47)$$

Substituting (46) into the self-consistency equation,

$$\hat{\Delta} \ln(t_c) = \pi T_c \sum_{n=-\infty}^{\infty} \left(i \hat{F}_s^S - \frac{\hat{\Delta}}{|\varepsilon_n|} \right); \quad t_c = \frac{T_c}{T_{c0}}, \qquad (48)$$

one finally gets the equation for finding the transition temperature of the dirty superconductor - strong ferromagnet bilayer:

$$\ln(t_c) = \Psi\left(\frac{1}{2}\right) - Re\,\Psi\left(\frac{1}{2} + \frac{\rho}{t_c}\right), \qquad (49)$$
$$\rho = \frac{D^S (\kappa^S)^2}{4\pi T_{c0}}.$$

4.3. Results and discussion of proximity effect

Upon solution of the equation (49) we will neglect dependence of κ^S on ε_n, because we consider strong ferromagnet with energy of the exchange splitting of conduction band which is much larger than the thermal energy. As is was done in the discussion of experiments on Andreev spectroscopy, we fix the Fermi momentum of the superconductor by the relation $\left(p^S\right)^2 = \frac{1}{2}\left[\left(p_\uparrow^F\right)^2 + \left(p_\downarrow^F\right)^2\right] = const$, and use the above formulas (27) to evaluate the interface transmission coefficients.

The results of calculations for the set of parameters: $\delta = p_\downarrow^F / p_\uparrow^F = 0.65$, $p_\uparrow^F l_\uparrow^F = 40.0$, $l_\downarrow^F / l_\uparrow^F = 2.5$, $\xi^S / \xi_{BCS}^S = 0.25$ [$\xi^S = (D^S / 2\pi T_{c0})^{1/2}$ – is the coherence length of dirty superconductor] are displayed in Fig. 3. The parameters approximately correspond to the contact of nickel with niobium or vanadium. The figure shows damped oscillations of transition temperature as a function of the ferromagnetic layer thickness. As the superconducting layer becomes thin enough, the re-entrant behavior of the superconducting transition temperature is possible (the lower, solid curve on Fig. 3), which has been observed in the experiment [46,47]. The results for the another calculation with only the exchange splitting parameter is changed, $\delta = p_\downarrow^F / p_\uparrow^F = 0.55$ (this corresponds approximately to cobalt), are displayed in Fig. 4. Comparison with the previous figure shows that superconducting T_c suppression is weakened in the contact with the stronger ferromagnet (δ(Co) < δ(Ni)), which seems to contradict expectation. However, one should keep in mind that when conduction band polarization growths, the interface transparency decreases as a result of increasing mismatch between Fermi momenta of the superconductor and the ferromagnet. Growing isolation of S and F layers dominates the increase of depairing influence of the exchange field. This scenario has been realized in the layered system $Fe_x V_{1-x} / V$ [48]. With increasing the iron content x in the ferromagnetic alloy $Fe_x V_{1-x}$ the non-monotonic behavior of the superconductor critical thickness was observed at fixed thickness of the ferromagnetic layer. The pure iron layer suppressed T_c weaker than the alloy with the iron concentration $x \simeq 0.6$. Our calculations take into account explicitly the dependence of the interface transparency on the conduction-band exchange splitting, giving the theoretical basis for the extensive discussions of the F/S interface transparency based on the experimental data [42,46-49]. Our results do not contradict conclusions by Bergeret *et al.* [22]: Figs. 1 and 2 show that at small T_c suppression, when $T_{c0} - T_c \ll T_{c0}$, and for $d^F > l_\uparrow^F$, the oscillations amplitude is considerably smaller than the asymptotic value of the suppression, $\delta T_c = T_{c0} - T_c(d^F \to \infty)$ (see upper curves in Figs. 3 and 4). Thus, the oscillations of T_c are beyond the approximation adopted in Ref. [22].

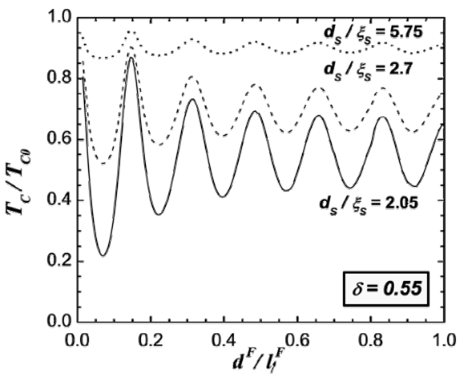

Fig. 3. Dependence of the superconducting critical temperature of F/S bilayer on the thickness of the ferromagnetic layer at δ = 0.65. Values of other parameters are given in the text

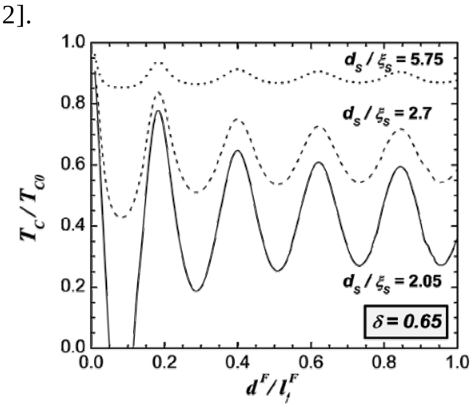

Fig. 4. Dependence of the superconducting critical temperature of F/S bilayer on the thickness of the ferromagnetic layer at δ = 0.55. Values of other parameters are the same as in Fig. 3

It interesting to note, that at certain thickness of the F-layer the decrement of T_c oscillations decay does not increase when decreasing the mean free paths l_α^F, as it could be expected, but decreases. This unusual behavior is explained by exclusion of slanted trajectories, along which the path of electrons inside the ferromagnetic film exceeds their mean free path. In the certain range of the thickness d^F the effect of closing the cone of effective trajectories, which couple F and S layers, dominates over their decay because of scattering. As the thickness dF approaches mean free paths, the cone of effective trajectories collapses toward the film normal, and solution of the problem approaches the single-exponential one [42].

5. Conclusion

In this paper we summarized results of our theoretical studies of the contacts of superconductors with strong ferromagnets. On the basis of microscopic approach we derived for the first time the Eilenberger-type equations of superconductivity for metals with exchange-split conduction band. The equations are valid for arbitrary band splitting and arbitrary spin-dependent electron mean free paths within the quasiclassic approximation. As a next step, we deduced general boundary conditions for the above equations. These BCs take into account explicitly the spin-dependence of F/S interface transparency. All other correct formulations of the boundary conditions for the F/S interface can be obtained as approximations, or limiting cases of our general BC.

We applied further our theory for the Andreev reflection at F/S interface and derived an original expression for the Andreev conductance. Our expression takes into account explicitly spin dependence of the interface transparency and spin-dependent conservation laws at scattering on the F/S interface. Based on the experimental data and our calculations we give estimations of the conduction-band spin polarization for series of ferromagnets in contact with superconductors.

Next, we considered the superconducting proximity for contact of strong and clean enough ferromagnet (electron mean free paths much longer than the oscillation period of the pairing function) with dirty (short mean free path) superconductor. We showed that superconducting T_c of the F/S bilayer oscillates as a function of the F-layer thickness. At small enough superconducting layer thickness the re-entrant behavior of superconductivity is predicted. The theory takes into account explicitly the spin dependence of the interface transparency, which results in non-monotonic dependence of the superconducting layer critical thickness on the spin polarization of the ferromagnetic layer. These unconventional and distinctive features of the F/S proximity effect fit qualitatively experimental observations.

Acknowledgments

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